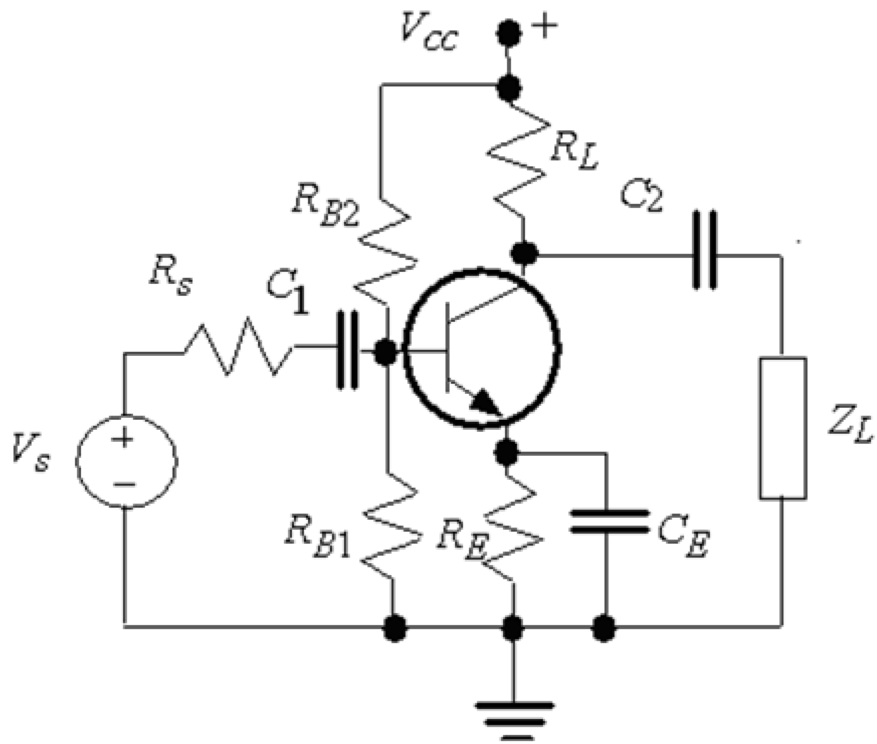
BJT high – frequency operation

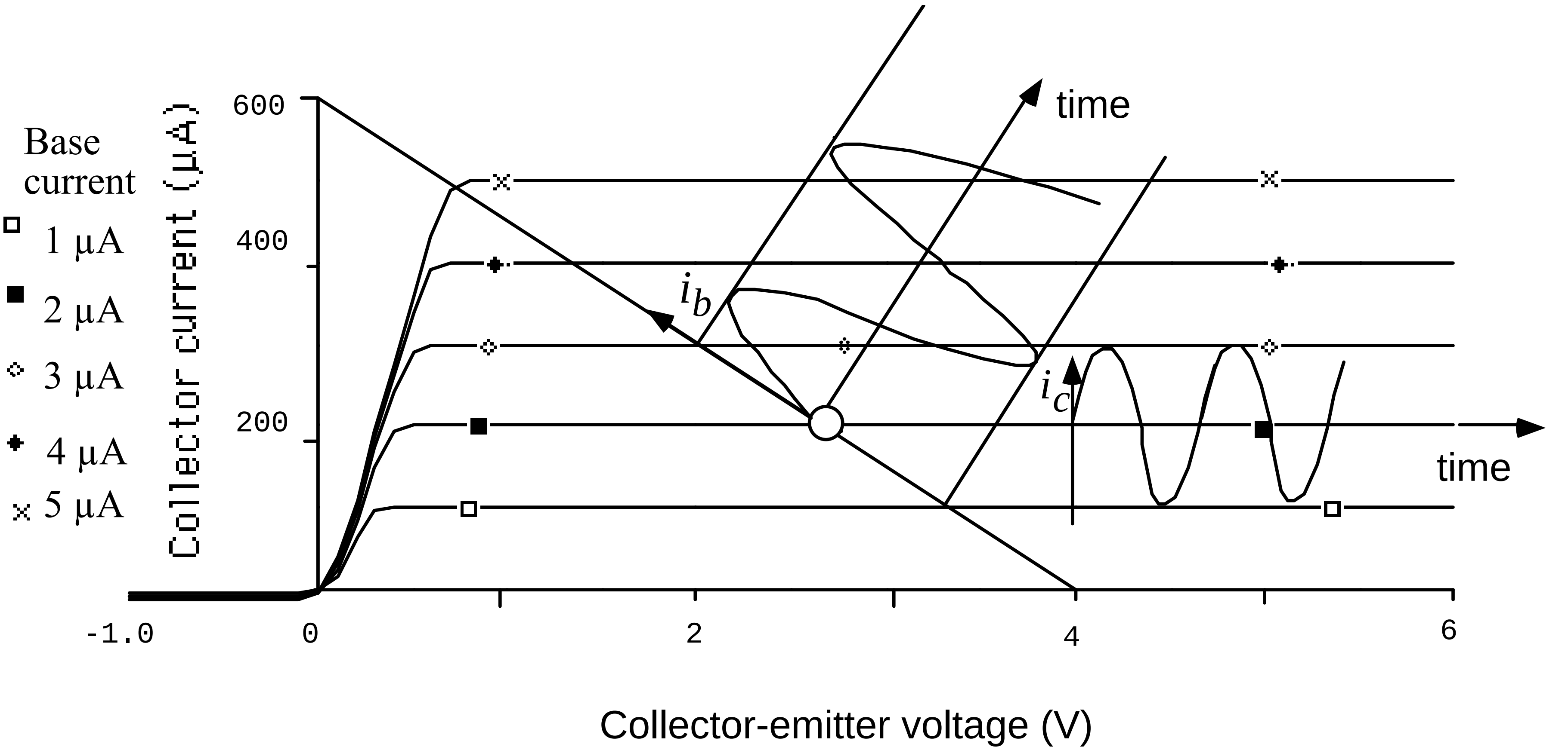
In practical common emitter amplifiers, capacitors C_1 and C_2 are used to block the DC current.

C_E acts are as bypass capacitor for shorting R_E at high frequency operation



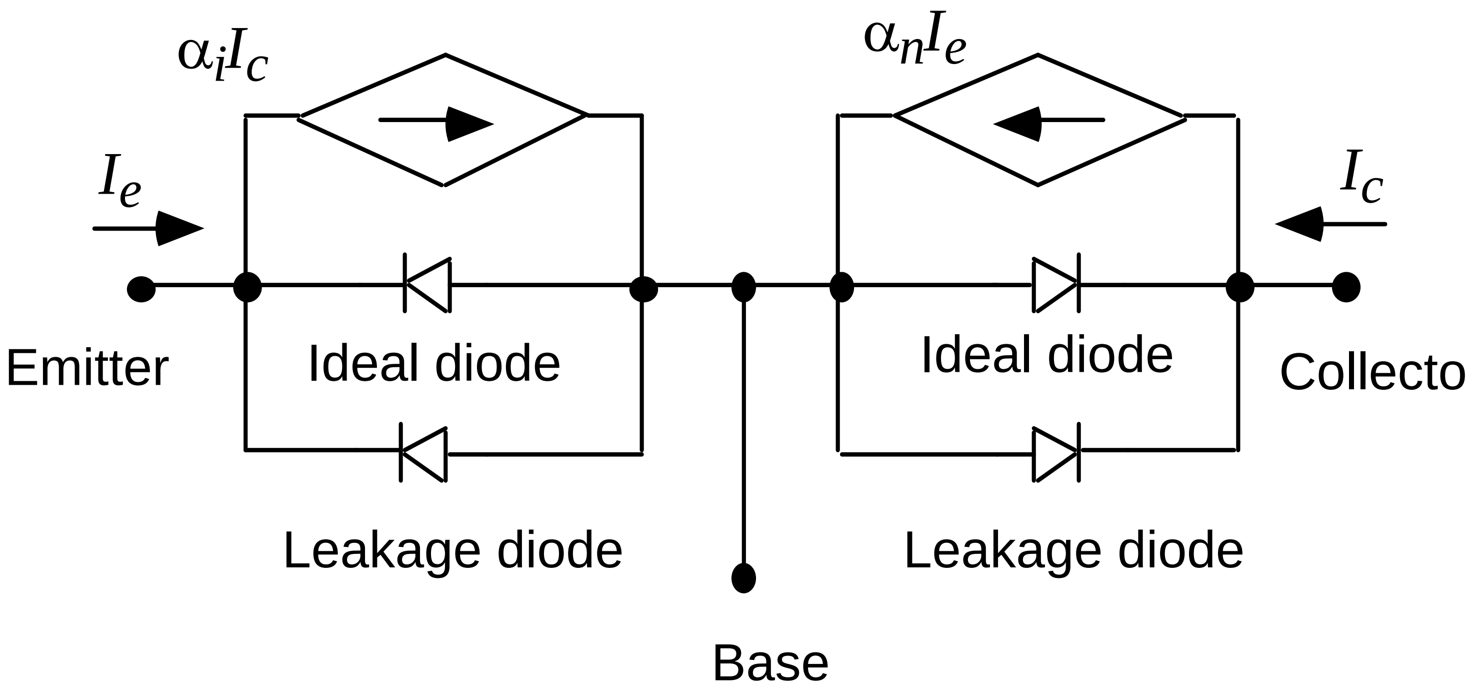
Consider medium frequencies, which are
(i) high enough so that the coupling and bypass capacitances have very small impedances and can be substituted by shorts
(ii) but are still much smaller than the BJT cutoff frequencies so that the internal transistor capacitances can be considered as open circuits.

BJT small-signal operation

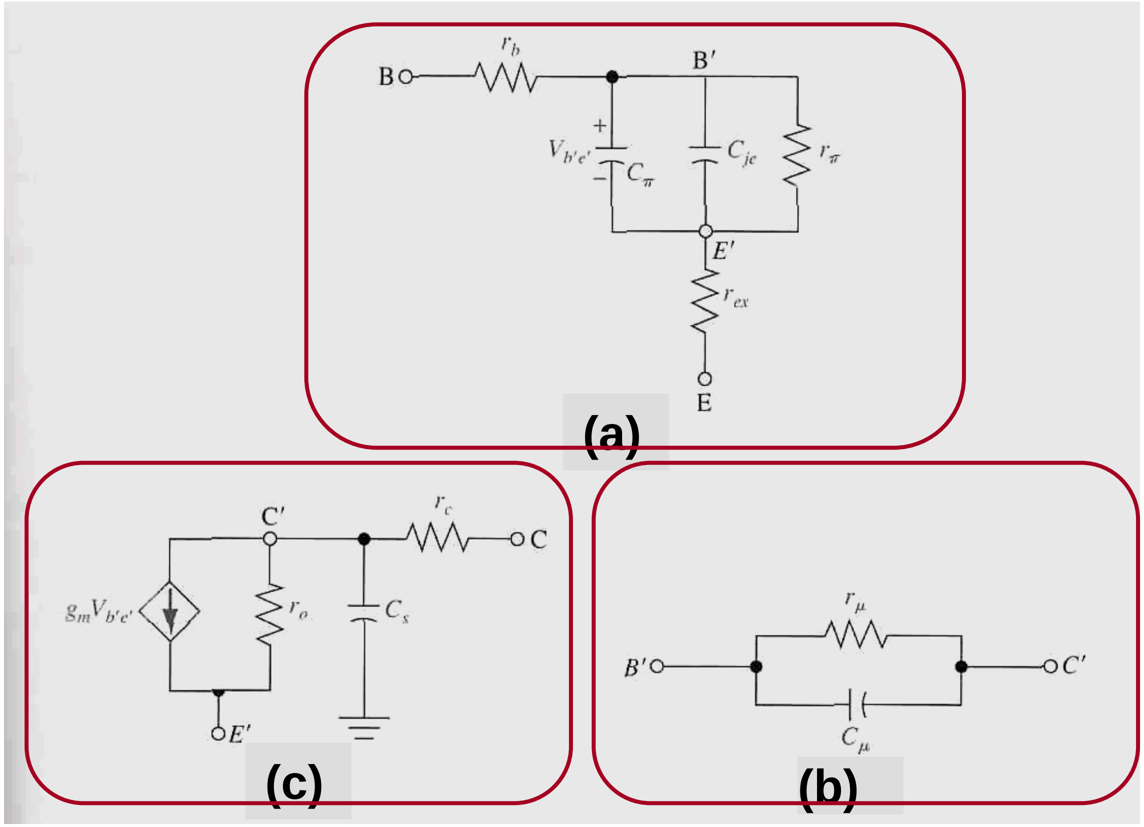


BJT high-frequency equivalent circuit

We start with the DC equivalent circuit:



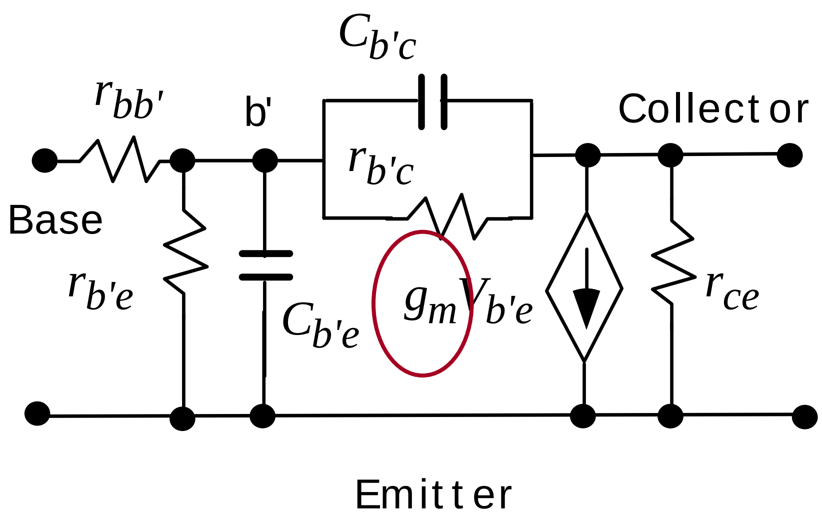
Additional high-frequency components of BJT terminals



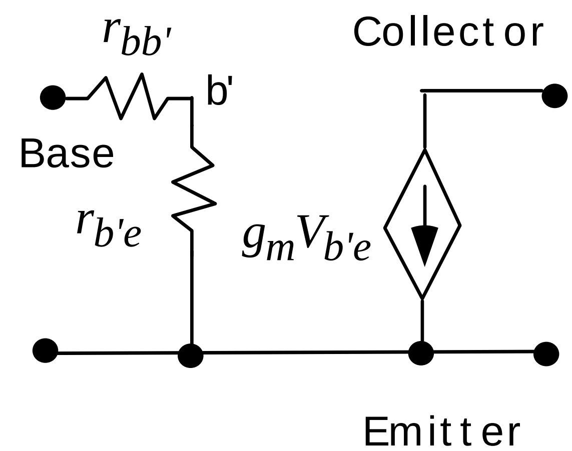
The base - emitter (a),
base-collector (b),
and collector – emitter (c)
components of the BJT equivalent circuit

Small-signal hybrid π model

Full



Simplified

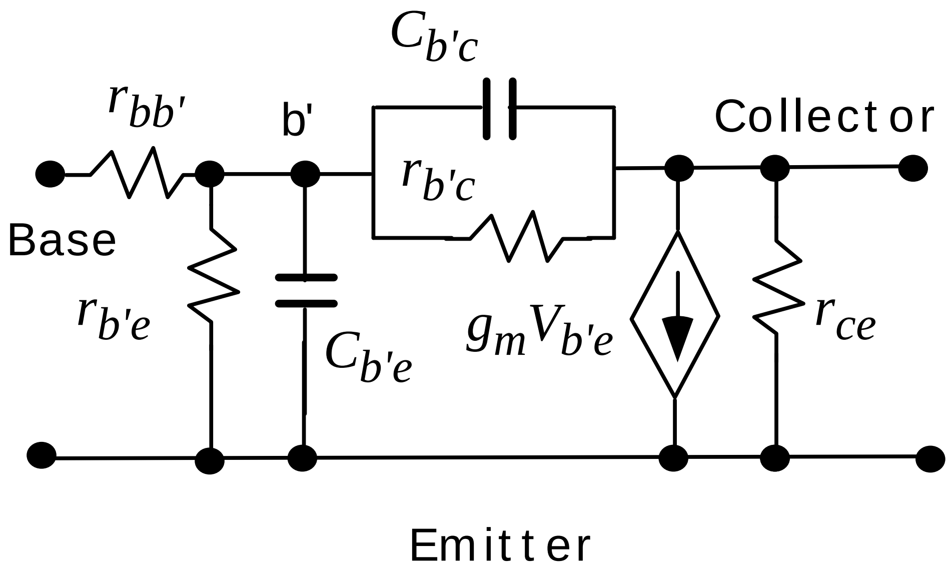


The **transconductance, g_m**, is related to the dynamic (differential) resistance, r_e, of the forward-biased emitter-base junction:

$$g_m = \frac{\partial I_c}{\partial V_{b'e}} = \alpha \frac{\partial I_e}{\partial V_{b'e}} \approx \frac{\alpha}{r_e} \approx \frac{I_c}{V_{th}}$$

$$V_{th} = k_B T/q$$

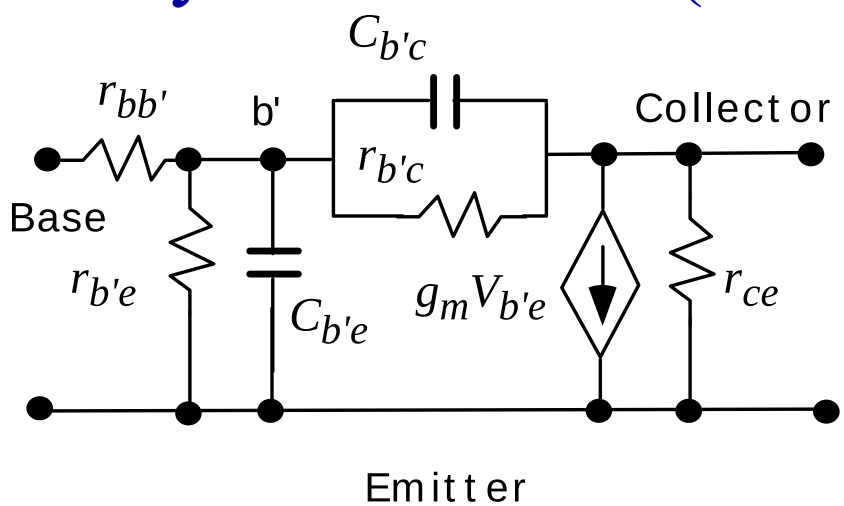
Hybrid π model (cont.)



The resistance $r_{bb'}$ is the base spreading resistance.

The resistance $r_{b'c}$ and the capacitance $C_{b'c}$ represent the dynamic (differential) resistance and the capacitance of the reverse-biased collector-base junction.

Hybrid π model (cont.)



Using transconductance: $i_c \approx g_m v_{b'e}$

(ignoring the current through r_{ce})

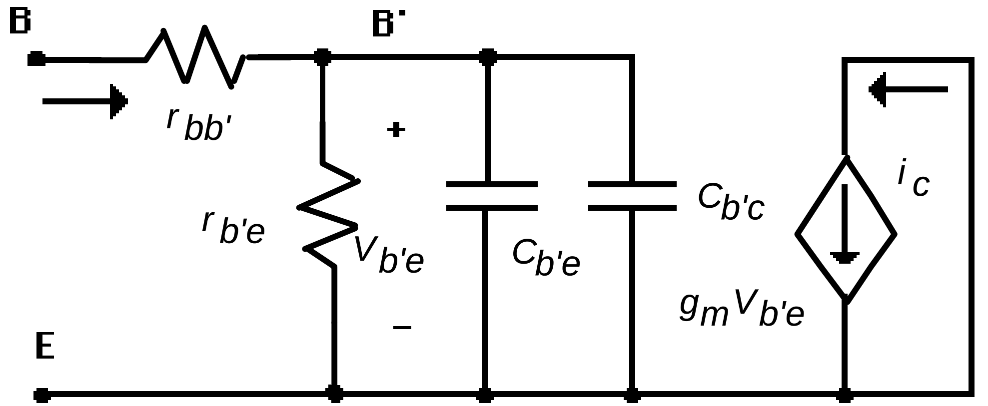
B-E differential resistance:

$$r_{b'e} \approx \frac{i_c}{i_b}\frac{1}{g_m} \approx \frac{\beta}{g_m}$$

B-E internal voltage
(ignoring the current through $C_{b'e}$):

$$v_{b'e} \approx i_b \, r_{b'e}$$

Frequency dependent gain (short-circuit load)



$v_{b'e} \approx i_b r_{b'e}$ transforms into $i_b = v_{b'e}\left[g_{b'e} + j\omega\left(C_{b'e} + C_{b'c}\right)\right]$

where, $g_{b'e} = 1/r_{b'e}$

Since $i_c = g_m v_{b'e}$, the short-circuit emitter current gain

$$\beta_\omega = \frac{i_c}{i_b} = \frac{g_m}{g_{b'e} + j\omega\left(C_{b'e} + C_{b'c}\right)} = \frac{\beta}{\left(1 + j\omega / \omega_\beta\right)}$$

where $\omega_\beta = 2\pi f_\beta = g_{b'e} / \left(C_{b'e} + C_{b'c}\right)$

The BJT cutoff frequencies

$$\beta_\omega = \frac{i_c}{i_b} = \frac{g_m}{g_{b'e} + j\omega\left(C_{b'e} + C_{b'c}\right)} = \frac{\beta}{\left(1 + j\omega / \omega_\beta\right)}$$

$$\omega_\beta = 2\pi f_\beta = g_{b'e} / \left(C_{b'e} + C_{b'c}\right)$$

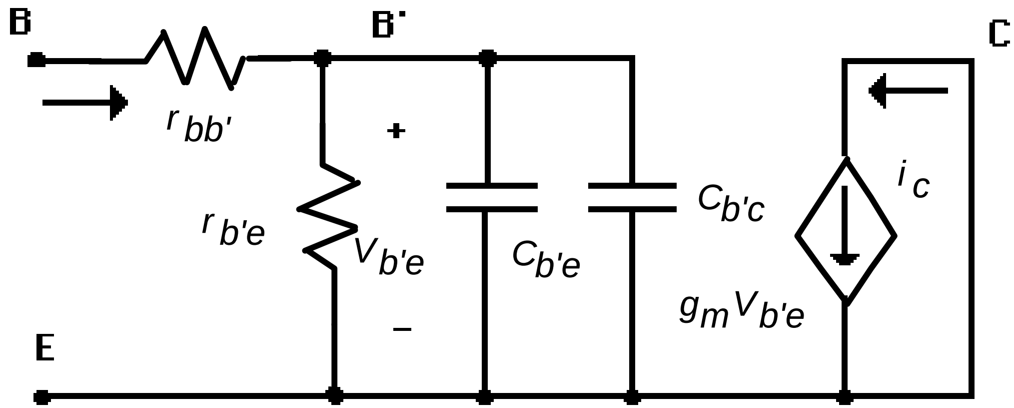
(1)Beta cutoff frequency f_β

is the frequency at which $\omega = \omega_\beta$

i.e. the magnitude of the common-emitter current gain decreases by a factor of $\sqrt{2}$

$$f_\beta = g_{b'e} / \left[2\pi\left(C_{b'e} + C_{b'c}\right)\right]$$

The BJT cutoff frequencies



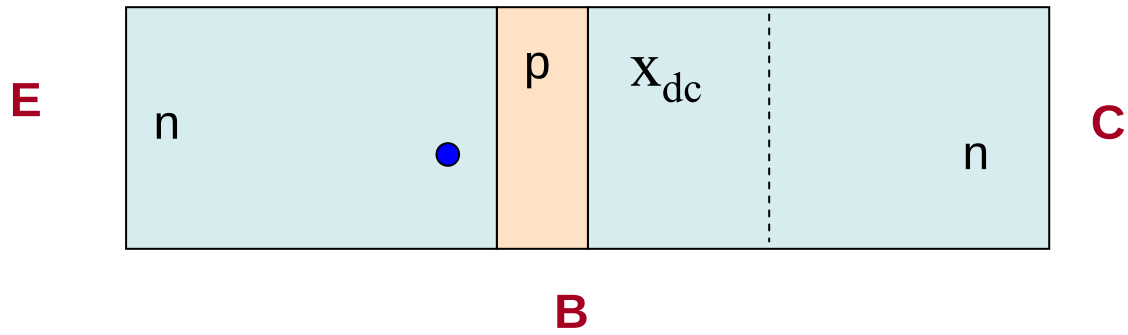
$$\beta_\omega = \frac{i_c}{i_b} = \frac{g_m}{g_{b'e} + j\omega\left(C_{b'e} + C_{b'c}\right)} = \frac{\beta}{\left(1 + j\omega / \omega_\beta\right)}$$

(2) Common emitter cutoff frequency f_T

is the frequency at which the magnitude of the common-emitter current gain equals unity, that is, $|\beta_\omega| = 1$.

$$f_T = f_\beta \sqrt{\beta^2 - 1} \approx \frac{g_m}{2\pi\left(C_{b'e} + C_{b'c}\right)}$$

Carrier transport delays in BJTs



Electrons injected into base diffuse across base and drift across the collector depletion region x_{dc} (typically x_{dc} is much larger then the base thickness W)

At high frequencies this transit time becomes comparable with the period of the input signal; at this point the output response will no longer be in phase with the input signal.

Carrier transport delays in BJTs (cont.)

Collector depletion region delay time:

$$\tau_{cT} \approx \frac{x_{dc}}{v_{sn}}$$

where x_{dc} is the width of the collector-base depletion region and v_{sn} is the electron saturation velocity (for *n*-*p*-*n* transistors).

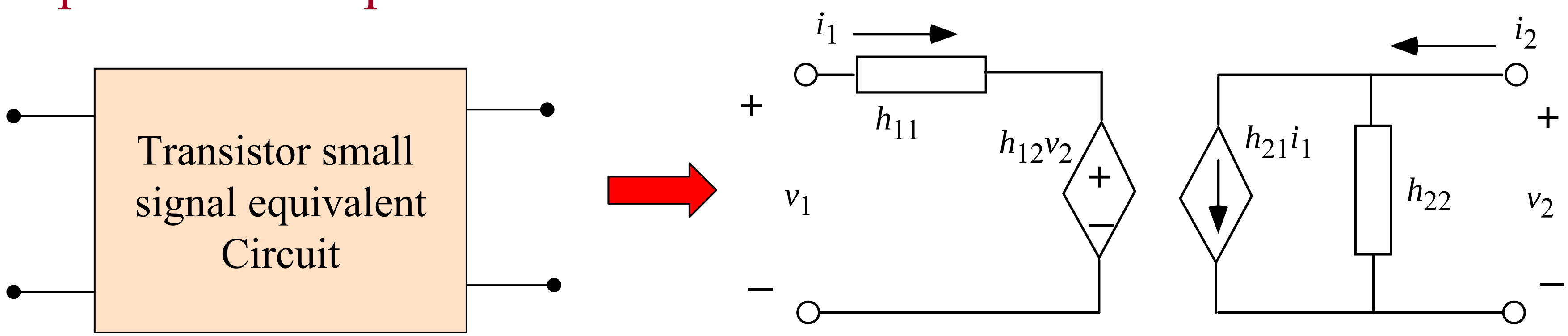
Other delays include collector and emitter charging times and base transit time

If the total BJT delay time is τ_{eff}, the cutoff frequency:

$$f_T \approx \frac{1}{2\pi\tau_{eff}}$$

Two-port network BJT equivalent circuit

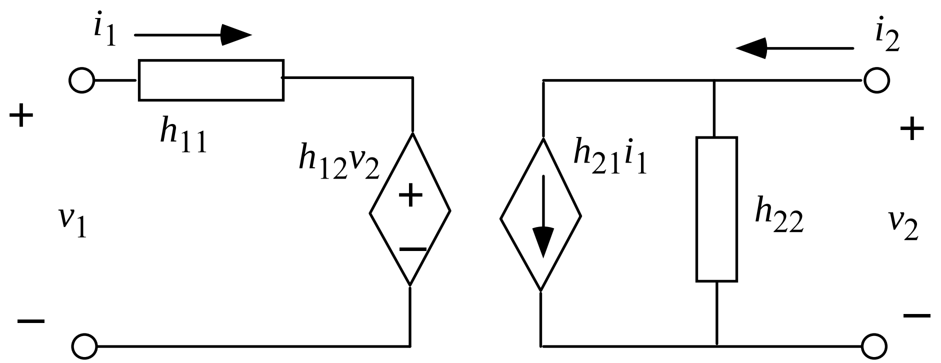
h-parameters equivalent circuit



$$v_1 = h_{11} i_1 + h_{12} v_2$$

$$i_2 = h_{21} i_1 + h_{22} v_2$$

h-parameters



short-circuit input impedance $$h_{11} = \left.\frac{v_1}{i_1}\right|_{v_2 = 0}$$

open-circuit reverse voltage ratio $$h_{12} = \left.\frac{v_1}{v_2}\right|_{i_1 = 0}$$

short-circuit forward current ratio $$h_{21} = \left.\frac{i_2}{i_1}\right|_{v_2 = 0}$$

open-circuit output admittance $$h_{22} = \left.\frac{i_2}{v_2}\right|_{i_1 = 0}$$

h-parameters and BJT parameters

Alternative notation:

*h*11 is also called the short-circuit input impedance (*hi*) ,

*h*12 is called the open-circuit reverse voltage ratio (*hr*),

*h*21 is called the short-circuit forward current ratio (*hf*), and

*h*22 is called the open-circuit output admittance (*ho*).

Second subscript is often used to denote the transistor configuration:

("e" for common emitter circuit).

h-parameter	Relation to parameters of hybrid π-equivalent circuit
h_{oe}	$1/\left(r_{b'c}+r_{b'e}\right)+1/r_{ce}+g_m r_{b'e}/\left(r_{b'c}+r_{b'e}\right)$
h_{ie}	$r_{bb'}+r_{b'e}r_{b'c}/\left(r_{b'e}+r_{b'c}\right)$
h_{fe}	$g_m r_{b'e}r_{b'c}/\left(r_{b'c}+r_{b'e}\right)$