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GenRad

A Precise Standard of High Capacitance

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The four-terminal capacitance standard described uses two autotransformers to scale the value of a 1-μF capacitor to higher decade values up to 1 F. The direct-reading accuracy is ±1/4% at 100, 120, or 1000 Hz. Much better accuracy is possible if the 1-μF value is precisely determined, because the scaling ratios are precise. Intermediate capacitance values can be obtained by using an external capacitor. The unit also acts as a standard of dissipation factor and is useful as a two-terminal capacitance standard up to 1000 μF at 100 or 120 Hz.

Introduction

Several instruments are available that measure high capacitance, some of them as high as 1 F, and standards are required to calibrate them. Stable capacitors that would make suitable standards would be extremely expensive at these values and quite impractical. This paper describes a method for precisely scaling a low-valued capacitance standard up to these high values and a particular device that uses this method.

All instruments that measure these high capacitance values are capable of making four-terminal measurements to avoid lead impedance errors. Therefore, four-terminal networks whose transfer impedances simulate high C values can be used for their calibration. Such a standard,[1] using a three-winding transformer, was available commercially but it had limited accuracy and frequency range. Several other networks have been suggested[2] and the most suitable of these is discussed here and is used in a new standard shown in Figures 1 and 2. It uses two, precise inductive voltage dividers to scale the capacitance of a 1-μF standard up to higher decade values.

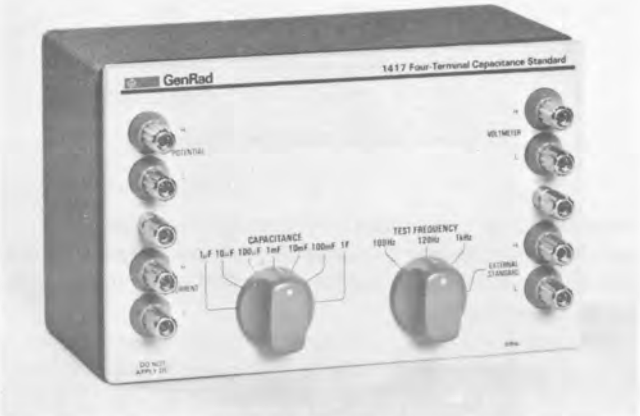

Figure 1. The 1417 Four-Terminal Capacitance Standard

Figure 2. Internal View Showing Construction (Transformer shields removed)

The Four-Terminal, or Transfer, Capacitance

This new standard uses the transfer impedance, E_O/I_{in}, of the network of Figure 3 as a standard of high capacitance. A laboratory implementation of this network would probably use two high-resolution, inductive voltage dividers (also referred to as IVD's or decade transformers). The new, self-contained standard uses two single-core autotransformers for lower cost, setting convenience, and some design advantages.

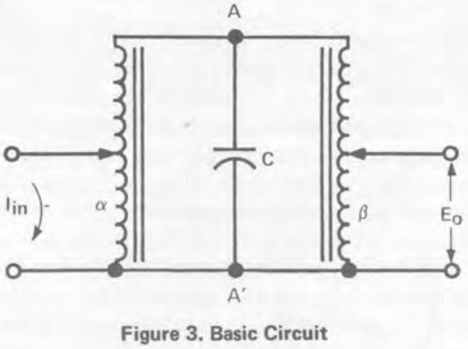

Figure 3. Basic Circuit

The input divider divides the input current by α and the output divider divides the voltage across the capacitor by β. If these dividers were ideal, that is, they had infinite open-circuit (magnetizing) impedance, zero short-circuit (winding) impedance and exact ratios, and if the capacitor were lossless, the transfer impedance would be purely capacitive with a value $C/\alpha\beta$. Moreover, if α and β were exactly equal and there were no wiring resistance or inductance, this network would act like a two-terminal capacitor of the same value. Needless to say, these ideal properties are not met, and the imperfection of each causes specific additions to such an ideal equivalent circuit.

An inductive divider is a passive, three-terminal network and can be represented by an equivalent, three-impedance Y network, as shown in Figure 4 (although it must be remembered that these impedances are non-linear).

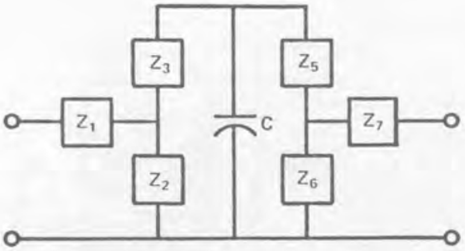

Figure 4. General Equivalent Circuit

For this network the transfer admittance is

$$Y_T = \frac{1}{Z_T} = \frac{I_{in}}{E_o} =$$

$$\left(\frac{Z_2+Z_3}{Z_2}\right)\left(\frac{Z_5+Z_6}{Z_6}\right) \times \left(j\omega C + \frac{1}{Z_2+Z_3} + \frac{1}{Z_5+Z_6}\right) \qquad (1)$$

The first two factors are the reciprocals of the open-circuit divider ratios α and β. Well-designed dividers, using high-permeability toroidal cores, can give ratios that are extremely precise and stable so that the ratio accuracies are not a practical limitation to the accuracy of the method. All the ratios measured were better than 50 ppm from 50 Hz to 2 kHz. Indeed, it is the strength of this method that the ratios between capacitance values depend only on these precise divider ratios, as long as the last factor can be kept precisely constant.

The last factor represents the admittance of the capacitor, C, in parallel with the open-circuit input impedances of the two dividers. These impedances are independent of the ratio settings, but do depend on frequency and signal level. If we assume for the moment that these input impedances are purely inductive and, in parallel, have a value of L, then this last factor becomes

$$j\omega C\left(1 - \frac{1}{\omega^2 LC}\right) = j\omega C' \qquad (2)$$

so that the transfer capacitance

$$C_T = \frac{Y_T}{j\omega} = \frac{C'}{\alpha\beta} = \frac{C}{\alpha\beta}\left(1 - \frac{1}{\omega^2 LC}\right) \qquad (3)$$

Thus the value of C′ depends somewhat on the test frequency and signal level. The LC product should be kept as large as possible. In the new standard, the difference between C and C′ is approximately 1/2% at 120 Hz. However, this frequency error is compensated at 100 Hz, 120 Hz, and 1 kHz by switched-in padding capacitors to give a direct-reading accuracy of 1/4% at these specific frequencies.

Calibrations can be made to much better accuracy by determining the value of C′, This can be done by measuring the capacitance between points A and A′, Figure 3, (terminals are available) with a two-terminal bridge, because this two-terminal value is equal to the four-terminal transfer capacitance when $\alpha = \beta = 1$. For best accuracy, a plot should be made of this capacitance versus signal level. Then, when higher values are measured, the voltage across the capacitor can be measured and the appropriate correction made. Accuracies of 0.05% and better are easily achieved.

It is desirable to have the lowest capacitance value be 1 μF so that the value of C′ can be precisely determined on a precision bridge or by comparison to a precise 1-μF standard. Available bridges and standards are substantially less accurate at higher capacitance values. If C = 1 μF, the value of L must be very high to make the difference between C and C′ small at 120 Hz. Most available IVDs have inductances in the 10-to-100 H range (giving a value of L of 5 to 50 H), which causes a rather large difference if C = 1 μF; it would be preferable to use a larger capacitor with these IVDs. For the new standard, the value of L is approximately 400 H. It could be even larger but this would result in excessive terminal impedance (see below).

Terminal Impedances

Although a true four-terminal measurement would be completely immune to impedance in series with any of the four terminals, no instrument makes such an ideal measurement, so these impedances should be kept small to avoid measurement errors.

The lower, two branch, or terminal, impedances of Figure 5 are small, for they are simply the impedance of the wired common connection of Figure 3. The upper two terminal impedances are much more critical. From the equivalent circuit of Figure 4, the (open-circuit) input impedance can be shown to be:

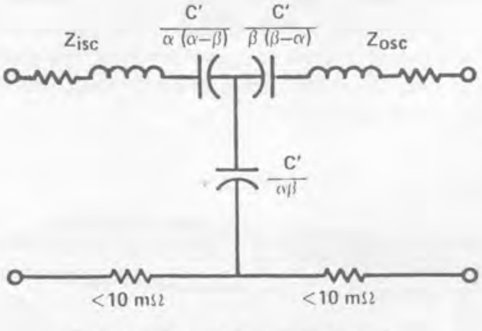

Figure 5. Actual Equivalent Circuit

$$Z_{in} = Z_1 + \frac{Z_2 Z_3}{Z_2+Z_3} + \left(\frac{Z_2}{Z_2+Z_3}\right)^2 \frac{1}{\left(j\omega C + \frac{1}{Z_2+Z_3} + \frac{1}{Z_5+Z_6}\right)} \qquad (4)$$

which can be written as:

$$Z_{in} = Z_{isc} + \frac{a^2}{j\omega C'} \qquad (5)$$

where Z_{isc} is the input impedance with a short across the capacitor. Subtracting the transfer impedance, we get the upper input terminal impedance

$$Z_{in} - Z_T = Z_{isc} + \frac{a^2}{j\omega C'} - \frac{\alpha\beta}{j\omega C'} = Z_{isc} + \frac{\alpha(\alpha-\beta)}{j\omega C'} \qquad (6)$$

(The other terminal impedance is easily found by symmetry.)

The first term is mostly resistive and a function of the winding resistance of the input divider. For a given core and winding area, this resistance is proportional to N^2, where N is the number of turns. Because the open-circuit inductance is also proportional to N^2, there is a tradeoff between keeping this resistance low and keeping the error of equation 2 small. The compromise reached was based on the characteristics of several instruments. These windings also have some leakage inductance, but the impedance of this inductance is small compared to the resistance, even at 1 kHz.

The second term in this terminal impedance is capacitive (+ or −) and is the result of inequality between the divider ratios α and β. These ratios can be exactly equal (nominally) when their product is 10^{-2}, 10^{-4}, or 10^{-6}, but, when it is 10^{-1}, 10^{-3}, or 10^{-5}, they can be equal only if they are irrational numbers. If these dividers are autotransformers (and not high resolution IVDs) and possible ratios are limited to ratios of integral numbers of turns, either these $\alpha\beta$ ratios must be slightly inaccurate or α and β must differ. A resistive divider across one turn of one divider can, however, give greatly increased resolution. Such a divider is used on the 10-μF setting because this capacitive, terminal impedance must be small to avoid an error on a specific bridge. The table below shows the winding scheme used and the resulting capacitive term of equation 6. The impedance of this capacitance is always much less than the winding resistance even at 50 Hz.

RATIO $1/\alpha\beta$	"CURRENT" TURNS	α	"POTENTIAL" TURNS	β	$\alpha\beta$ ERROR	$\frac{C'}{\alpha(\alpha-\beta)}$
1	3000	1	4000	1	0	∞
10	949	.316333	1264½*	.316125	+9 ppm	15 mF
10^2	300	0.1	400	0.1	0	∞
10^3	96	.0320	125	.03125	0	42 mF
10^4	30	.01	40	.01	0	∞
10^5	10	.003333	12	.003	0	.9 F
10^6	3	.001	4	.001	0	∞

*Resistive divider across one turn

$\sqrt{10} = 3.16227766\ldots.$

High-Frequency Behavior

While the parallel inductance effect of equation 3 becomes negligible at higher frequencies, other factors eventually limit the accuracy as the frequency is increased. First, the divider ratio accuracy deteriorates above 2 kHz; errors approach 0.1% at 10 kHz where they are increasing as f^2.

A second, more limiting, effect is that of mutual inductance between the input ("current") and output ("potential") circuits. The relationship is

$$C'_T = \frac{C_T}{1 \pm \omega^2 M C_T}$$

where C_T is the transfer capacitance ($C'/\alpha\beta$) and M is the mutual inductance which can be positive or negative. There is some coupling external to the standard as well as internal. The external mutual inductance between connecting leads is most important for very high values of C_T, even at low frequencies. For example, only 2.5 nH causes a 0.1% error when measuring 1 F at 100 Hz or when measuring 10 mF at 1 kHz. It takes great care in making connections to keep the mutual inductance below this value. At 1 kHz, accurate measurements above 10 mF are rather impractical.

In spite of magnetic shielding and careful wiring, the internal mutual inductance does have a slight effect at 1 kHz, an error of about 0.01% even at lower C values.

Other Characteristics

The new standard provides decade values of capacitance to 1 F with its internal standard, but an external capacitor may be used to obtain intermediate or higher values. This capacitor may be added in parallel with the internal standard or used by itself. When added to the internal standard, the compensation for the parallel L error (equation 3) is still valid.

The dissipation factor, D, of the standard is intentionally made to be 0.01 at 100, 120 and 1 kHz so that, when measured on a bridge or meter with slight D error, the D reading will be positive. The accuracy of D is ±0.001 at these frequencies so that the device is a good standard of D as well as of capacitance. D errors can be caused by phase errors in the divider ratios and by loss in the open-circuit divider impedance, but both these effects are small.

The standard can be used as a two-terminal standard as long as higher D values can be tolerated. The D is less than 1 and the capacitance error is less than 1/2% up to 1000 μF at 120 Hz. This standard could be used to calibrate the higher capacitance ranges of the popular "universal" or "CRL" bridges.

Conclusions

The standard described has proven useful in the calibration of several existing instruments and should be adequate to meet the needs of new instruments for many years. One feature about the device is that all its parameters are measurable (without disassembly) so that, in effect, its ultimate accuracy depends on the accuracy of external measurement equipment.

References

[1] H.P. Hall, "Standards of Large Capacitance," *GR Experimenter,* January 1968.

[2] H. P. Hall, "Standardization of a Farad," *Electronic Instrument Digest,* August 1970.

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