Frequency Behavior of Precision Resistors

Basic Resistor Equivalent Circuit, Effects of L and C.
The basic lumped-parameter equivalent circuit for a two-terminal resistor is that shown here. For this circuit:

```
      R      L
 O+-/\/\/\-ωωωωω-+o
  !              !
  +------)(------+
         C
```

$$Z = \frac{R + j\omega[L(1-\omega^2 LC) - R^2 C]}{(1-\omega^2 LC)^2 + (\omega RC)^2} = Rs + jXs \qquad [1]$$

$$Y = \frac{R + j\omega[R^2 C - L(1-\omega^2 LC)]}{R^2 + (\omega L)^2} = Gp + jBp = \frac{1}{Rp} + jBp \qquad [2]$$

From these formulas we can get the equivalent series resistance, Rs, and the equivalent parallel resistance, Rp (the reciprocal of the real part of Y).

$$Rs = \frac{R}{1 - \omega^2 C(2L - R^2 C) + (\omega^2 LC)^2} \cong \frac{R}{1 - 2\omega^2 CL + (\omega RC)^2} \qquad [3]$$

as $(\omega^2 LC)^2$ is very small compared to $2\omega^2 LC$ unless $\omega^2 LC$ is large,
and

$$Rp = R[1 + (\omega L/R)^2] \qquad [4]$$

and also $Q = Xs/Rs = -Bp/Gp = \omega[L(1 - \omega^2 LC) - R^2 C]/R \cong \omega(L/R - RC)$ [5]

Note that Rs, the equivalent series value, is quite dependent on the lumped shunt capacitance if R is of high value, whereas Rp is independent of lumped shunt capacitance but quite dependent on series inductance if R is of low value. Thus Rs should be measured to get a more accurate measure of the dc value of low-valued resistors and Rp should be used for high-valued resistors. Of course a low frequency should be used in either case.

Note also that when referring to resistance at ac, one usually means the equivalent series resistance, Rs, unless otherwise specified. The relationship between Rs and Rp is

$$Rp = Rs(1 + Q^2) \qquad [6]$$

Dielectric Loss in the Lumped Shunt Capacitance
If the lumped parallel capacitance has loss, the resistor is shunted by conductance of ωDC. This affects both the effective series and parallel values of resistance but Rs would be much more affected by the capacitance itself.

```
          R
 o-+-/\/\/\/\/\/\-+-o
   !      C      !
   +--+--)(--+----+
      !      !
      +-/\/\-+   G = ωDC
```

$$Rp = R/(1 + RG) = R/(1 + \omega DCR) \qquad [7]$$

This effect can greatly decrease the ac resistance of high-valued resistors. For example if the capacitance across the resistor is 10 pF and has a D of .01, then the decrease in a 100 kΩ resistor at 1 MHz is 6.3%.

Note that the error is not proportional to frequency squared as are errors represented by lumped C and L parameters. However it is not exactly proportional to frequency as suggested by the above equation because D itself is not constant with frequency, although it usually varies rather slowly with frequency.

Capacitance to Guard (which is often Ground)

If a guarded (3-terminal) measurement is made and there is capacitance from the body of the resistor to guard, it produces an effective series inductance. If this capacitance is evenly distributed along the resistor, to the first order it can be approximated by a lumped capacitance, Cg, from the center of the resistor to guard, Cg, that is 1/2 of the total capacitance to guard, C. This produces an effective series inductance is $CgR^2/4 = CR^2/8$ which adds to the actual series inductance in equations [4]. If that real inductance is negligible, we have

```
      R/2    R/2
o-/\/\/\+/\/\/\/-o
        |
       ---
       ---  Cg =(1/2)C
        |
  Guard O
```

$$Rp \cong R[1 + (\omega RCg)^2/16] = R[1 + (\omega RC)^2/64] \quad [8]$$

$$Q \cong \omega CgR/4 = \omega CR/8 \quad [9]$$

[The remaining part the total capacitance is from the terminals to guard and does not affect a guarded measurement.]

This effective inductance adds to the actual inductance in the equation for Rs in equation [3] above. (Note Q is positive in equation [9].)

This is usually has only a small effect but can be important for physically-long, high-valued resistors near a guard or even in “free space”.

Dielectric Loss in the Capacitance to Guard

If the capacitance to guard has loss, it effectively causes an increase in both Rp and Rs. If we let Dg be the dissipation factor of the stray capacitance Cg, we get a conductance $Gg = \omega DgCg$ from the middle of the resistor to guard (if we assume that simple model). This causes an increase in both Rp and Rs.

$$Rp \cong R[1 + \omega DgRCg/8] \quad [10]]$$

Distributed Capacitance Along Resistor

Resistors also have capacitance between all points along its body to all other points. Modeling depends on the geometry of the resistor and is difficult, but the effect is similar in form, though not in value, to that of a single capacitance, Cy, shunting part of the resistor as shown.

```
  (1-α)R    αR
o-/\/\/\-+-/\/\/\-+o
         |        |
         +--)(----+

            Cy
```

$$Rp \cong R[1 - \alpha^3(1-\alpha)(\omega RCy)^2 \quad [11]$$

$$Rs \cong R[1 - \alpha^3(\omega RCy)^2] \quad [12]$$

$$Q \cong - \alpha^2\omega RCy \quad [13]$$

This is the classic cause of the decrease of resistance with frequency of high-valued resistors. There is always less error in Rp than in Rs for this effect.

Note that, if equal capacitances are in series shunting equal parts of the resistance, then these capacitances are equivalent to a very small lumped capacitance across the whole resistance and would not affect Rp. For some types of resistors, many of the individual capacitances may be treated this way leaving only the remaining capacitances in the situation in the figure above.

The effect of distributed capacitance is particularly important in shielded resistors when the shield is tied to one end of the resistor as in the case of the coaxial GR 1442 resistors when one end is shorted to make the resistor two-terminal. If the capacitance is evenly distributed along the resistor,

$$Rp \cong R[1 - (\omega RC)^2/45] \quad [14]$$

$$Rs \cong R[1 - 2(\omega RC)^2/15] \quad [15]$$

where C is the total capacitance from the resistor itself to the outer conductor. There is other capacitance in the connectors and resistor supports that at one end shunts the whole resistor and at the other is shorted.

These capacitances can be internal to the resistor. High-valued, carbon-composition resistors are subject to the "Boella Effect", the error caused by capacitances between the granules of carbon inside the resisters.

Eddy Current Loss

If a wire-wound resistor is wound on a conducting form or a resistor is in a conducting case, eddy currents in the metal will cause a power loss increasing the resistance as frequency increases. This can be represented by resistance shunting the resistor's effective lumped series inductance. If we call the shunting resistance Rv, we get

$$Rs = R[1 + \omega^2 L^2 (Rv/R)/(Rv^2 + \omega^2 L^2)] \quad [16]$$

A better model is mutual inductance, M, to a series combination of resistance, Re, and inductance, Le, that represents loops in the nearby metal. This gives

$$Rs = R[1 + \omega M^2 (Re/R)/(Re^2 + \omega^2 Le^2] \quad [17]$$

For both models there may be such several error terms representing eddy current loss in different parts of the conducting material. While the resistances increase with frequency, the inductance decreases lowering the Q of the resistor.

This effect is usually seen only in low-valued resistors. An extreme case is the resistance of iron-cored coils or transformers which vary widely with frequency due to eddy-current and hysteresis losses.

"Skin Effect" is due to eddy currents in the resistor itself. It causes an increase in resistance in thick, low-resistance conductors at high frequencies but is usually not seen in precision resistors other than low-valued current shunts.

Combining Errors

Because we are only interested in each type of error when it is small, the error for each can considered separately and the different errors may simply be added. Because each is small, their products would be very small, second-order error effects.

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