[Limitations of two-port analysis of negative feedback circuits]

[Sergio Franco] - May 03, 2018

Editor's note: I am posting this excellent tech article on EDN so that you will all be aware of Professor Sergio Franco's blog on EDN entitled [Analog Bytes]. This blog has some really great engineering analyses that provide an in-depth insight to engineers for their work.
—[Steve Taranovich]

Two-port (TP) analysis is widely used in the study of negative-feedback circuits. This type of analysis requires that we first identify which of the *four topologies* the circuit at hand belongs to (series-shunt, shunt-series, series-series, or shunt-shunt), and then that we suitably *modify* the basic amplifier so as to account for *loading* by the feedback network [1]. Textbooks do specify that TP analysis postulates certain approximations so that its results are not necessarily *exact*. However, not many textbooks dwell further into this issue by showing actual examples [2] for which TP analysis is insufficient, if not utterly inadequate; so, after becoming proficient in TP analysis, one may erroneously be tempted to take its results as *exact*. Let us illustrate using the voltage follower of **Figure 1** as a vehicle.

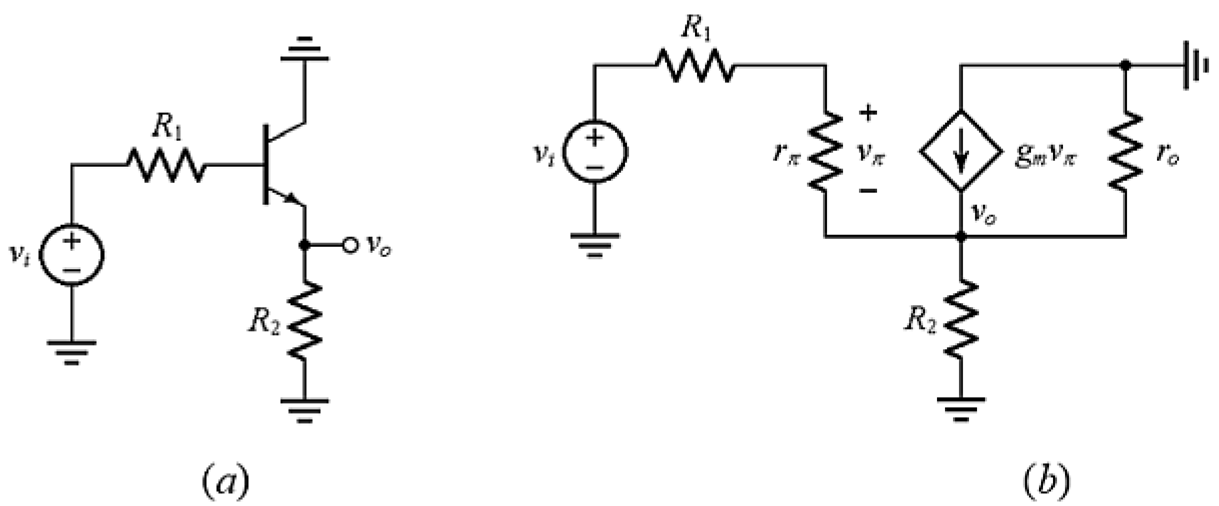

Figure 1 (*a*) Voltage follower and (*b*) its small-signal equivalent. Assume g_m = 20 mA/V, r_π = 2.5 kΩ, r_o = 50 kΩ, and $R_1 = R_2$ = 1.0 kΩ.

The voltage follower of **Figure 1*a*** forms a *series-shunt* configuration, and it is simple enough that we can find its **exact closed-loop gain directly**. With reference to its ac equivalent of **Figure 1*b***, we use nodal analysis and readily find the familiar expression [2]:

$$A_{exact} = \frac{v_o}{v_i} = \frac{1}{1 + \frac{R_1 + r_\pi}{(1 + g_m r_\pi)(R_2 // r_o)}} \quad (1)$$

Next, we use **TP analysis** to put the circuit in the block-diagram form of **Figure 2*a***, where a_{mod} is the gain of the basic amplifier *modified* so as to take loading into account, and A_{ideal} is the closed-loop gain in the limit $a_{mod} \to \infty$ The gain of this circuit takes on the familiar form [2]:

$$A_{TP} = \frac{v_o}{v_i} = A_{ideal} \frac{1}{1 + A_{ideal} / a_{mod}} \quad (2)$$

where the ratio a_{mod}/A_{ideal} is also called the *loop gain*. With reference to **Figure 2*b***, we write, by inspection,

$$v_o = g_m v_\pi (R_2 // r_o) = g_m (R_2 // r_o) \frac{r_\pi}{R_1 + r_\pi} v_i$$

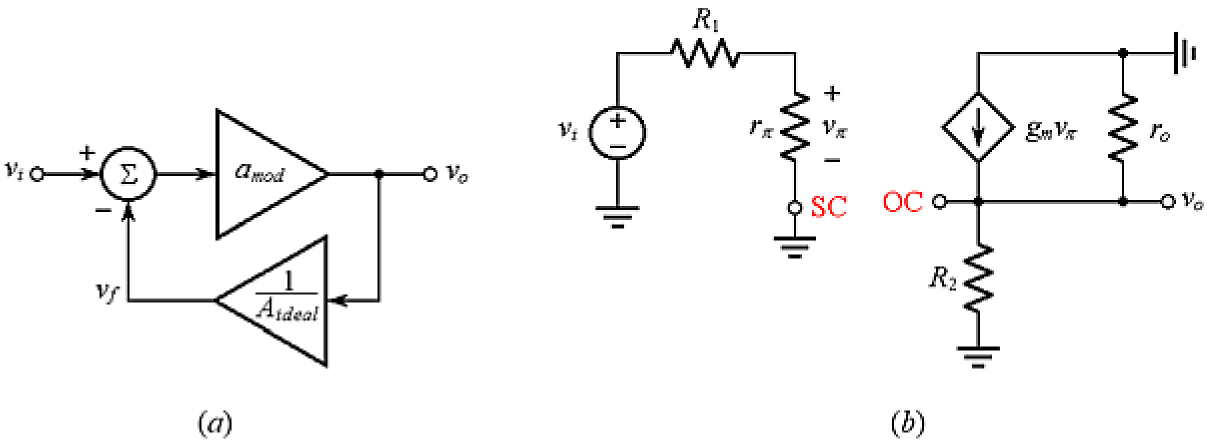

Figure 2 (*a*) Block-diagram for TP analysis, and (*b*) circuit to find the modified gain $a_{mod} = v_o/v_i$.

$$a_{mod} = \frac{v_o}{v_i} = g_m r_\pi \frac{R_2 // r_o}{R_1 + r_\pi} \quad (3a)$$

The condition $a_{mod} \to \infty$ needed to find A_{ideal} is achieved by letting $g_m \to \infty$. This results in $v_\pi \to 0$, so the current through r_π (and, hence, through R_1) tends to zero, indicating that $v_o \to v_i$. Consequently,

$$A_{ideal} = 1 \text{ V/V} \tag{3b}$$

Substituting a_{mod} and A_{ideal} into Equation (2), we get

$$A_{TP} = \frac{v_o}{v_i} = \frac{1}{1 + \dfrac{R_1 + r_\pi}{(0 + g_m r_\pi)(R_2 // r_o)}} \tag{4}$$

where “0” has been deliberately shown in the expression for A_{TP} to contrast it with the corresponding “1” appearing in the expression for A_{exact} above. Using the component values of **Figure 1**, we get

$$A_{exact} = 0.93458 \text{ V/V} \qquad A_{TP} = 0.93336 \text{ V/V} \tag{5}$$

The difference is minimal in the present case, which pertains to low-frequency operation, but it becomes much more pronounced at high frequencies, as we are about to show.

To investigate the frequency behavior, we use the ac equivalent of **Figure 3**, which includes the base-emitter capacitance C_π, the parasitic element dominating the emitter follower's dynamics. The expressions of Equations (1) and (4) still hold, provided we make the substitution

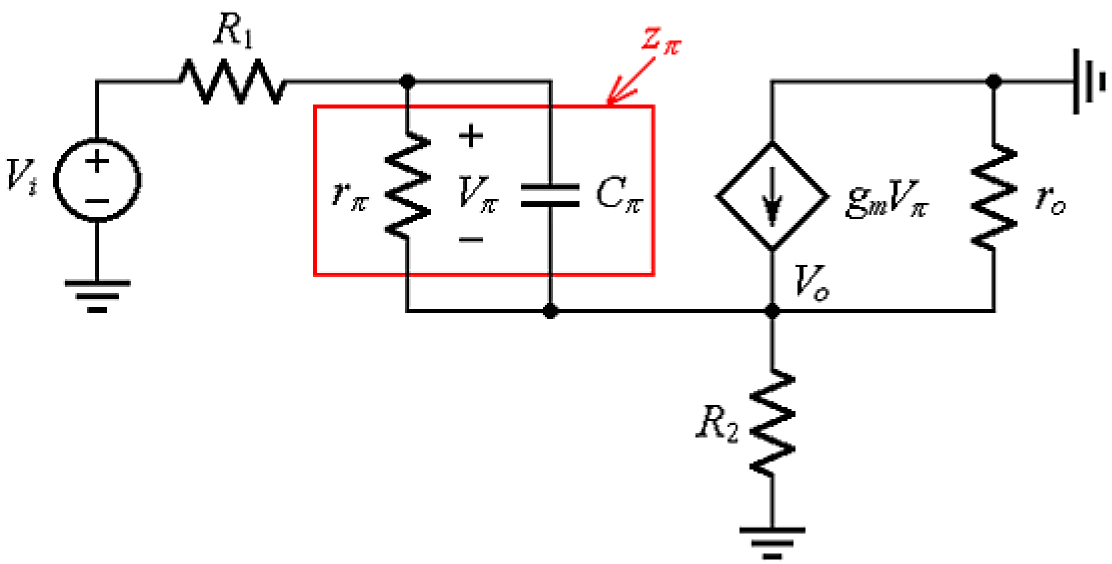

Figure 3 Including the capacitance C_π to investigate the frequency response

$$g_m r_\pi \to g_m z_\pi = g_m\left[r_\pi // (1/j\omega C_\pi)\right] = \frac{g_m r_\pi}{1 + j\omega r_\pi C_\pi} \tag{6}$$

after which both A_{exact} and A_{TP} become functions of $j\omega$. Now, considering that for $\omega \to \infty$ we have $g_m z_\pi(j\omega) \to 0$, *it follows that*

$$\lim_{\omega\to\infty} A_{exact}(j\omega) = \frac{R_2 // r_o}{R_1 + r_\pi + R_2 // r_o} \tag{7a}$$

$$\lim_{\omega\to\infty} A_{TP}(j\omega) = 0 \tag{7b}$$

Mathematically, the dramatic departure of A_{TP} from A_{exact} stems from the aforementioned denominator term of "0" instead of "1". Physically, we justify by noting that at high frequencies, where C_π approaches short-circuit behavior, both V_π and $g_m V_\pi$ will approach zero, thus reducing the circuit of **Figure 3** to a mere voltage divider, as confirmed by Equation (7*a*). Evidently, TP analysis fails to account for this physical reality, and therefore its results must be taken with a grain of salt.

An elegant alternative to TP analysis, and one that yields *exact* rather than approximate results, is ***return-ratio* (RR) analysis**. This type of analysis is based on the bock-diagram of **Figure 4**, where T is the *return ratio* of the dependent source modeling the amplifier's gain, and a_{ft} is the *feedthrough gain*, that is, the gain with the dependent source set to zero. The gain of this circuit takes on the form [2]

$$A_{RR} = \frac{v_o}{v_i} = \frac{A_{ideal}}{1 + 1/T} + \frac{a_{ft}}{1+T} \tag{8}$$

To find T (also called the loop gain) and a_{ft} for our voltage-follower example, refer to the ac equivalents of **Figure 5**. In **Figure 5*a*** we apply a test current i_t and find the return current i_r as

$$i_r = g_m v_\pi = g_m \frac{r_\pi}{R_1 + r_\pi}(-v_e) = -g_m \frac{r_\pi}{R_1 + r_\pi}\left[(R_1 + r_\pi) // R_2 // r_o\right] i_t$$

The return ratio T of the $g_m v_\pi$ *source is then [1, 2]*

$$T = -\frac{i_r}{i_t} = g_m v_\pi = g_m \frac{r_\pi}{R_1 + r_\pi}\left[(R_1 + r_\pi) // R_2 // r_o\right] \tag{9a}$$

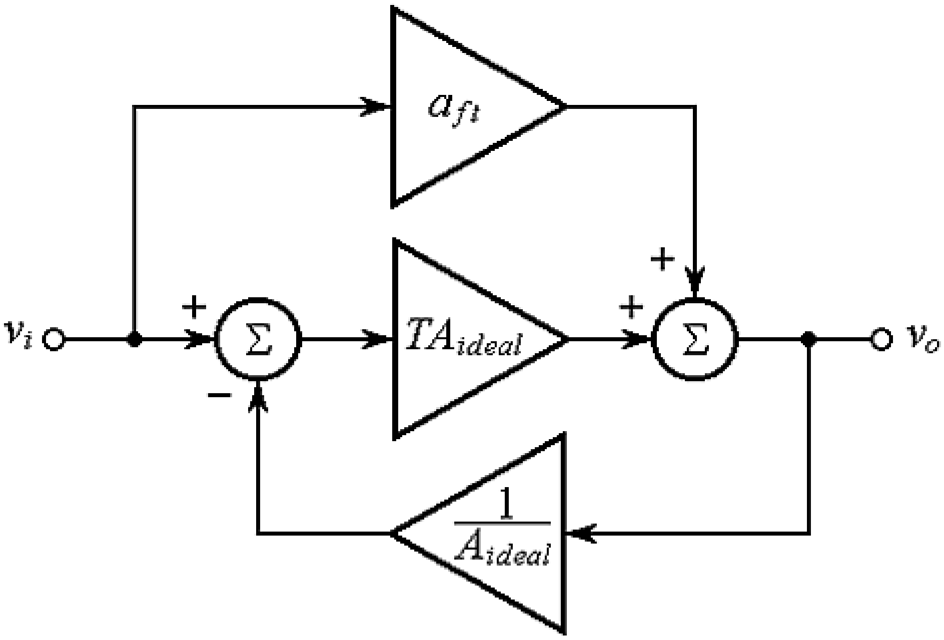

Figure 4 Block-diagram for RR analysis.

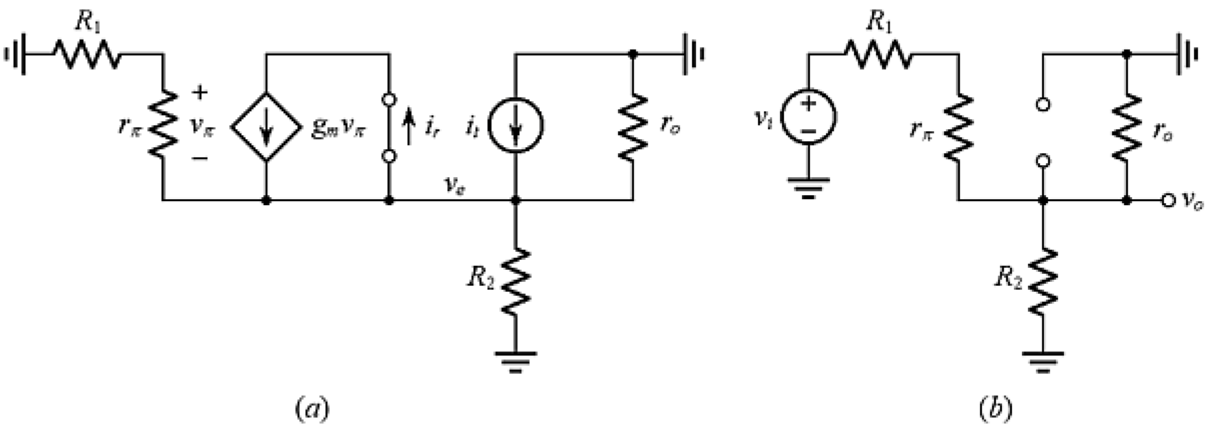

Figure 5 Circuits to find (*a*) the return-ratio *T* of the source $g_m v_\pi$*, and (b) the feedthrough gain* a_{ft} *around the same source.*

In **Figure 5*b*** we use the voltage divider formula to write

$$a_{ft} = \frac{v_o}{v_i} = \frac{R_2 // r_o}{R_1 + r_\pi + (R_2 // r_o)} \tag{9b}$$

Calculating *T* and a_{ft} with the component values of **Figure 1** and plugging into Equation (8) gives

$$A_{RR} = 0.91625 + 0.01833 = 0.93458 \text{ V/V} \tag{10}$$

which *coincides* with the value of A_{exact} of Equation (5). In fact, substituting Equation (9) into Equation (8), one can verify, with a bit of algebraic manipulations, that the expression of A_{RR} *coincides* with the expression of A_{exact}. But, from a bookkeeping viewpoint, A_{RR} is a bit more instructive because it shows separately the component due to forward gain and that due to feedthrough.

The case of the current-feedback amplifier

The case of the current-feedback amplifier (CFA)

If there is a circuit that has suffered significantly from the limitations of TP analysis, and as such has been the subject of unjust accusations – that circuit is the current-feedback amplifier (CFA). This elegant device has been the subject of two previous blogs [3,4], so here only its salient features are summarized.

With reference to **Figure 6**, top, Q_1 though Q_4 and Q_{11} though Q_{14} form two *identical* unity-gain voltage buffers with low output resistances r_n and r_o, respectively (see **Figure 6**, bottom). Any current imbalance I_n induced by the external circuitry (not shown yet) at the inverting input node is sensed and replicated by the pair of complementary Wilson current mirrors Q_5 though Q_7 and Q_8 though Q_{10} to the input node of the output buffer, also called the *gain node* (again, see **Figure 6**, bottom). In a well-designed CFA, this node exhibits a *large* equivalent resistance R_{eq} and a *small* parasitics capacitance C_{eq}. To get an idea, we have

$$R_{eq} = r_{c7} \,//\, r_{c8} \,//\, r_{b14}$$

where r_{c7} and r_{c8} are the output resistances of the Wilson mirrors, and r_{b14} is the input resistance of the output buffer. Considering that the output resistance of a Wilson mirror [2] is $(1 + \beta/2)r_o$, where r_o is the collector's ac resistance of the output BJT, and that the input resistance of a Darlington-type buffer such as the one shown is on the order of β^2 times the output load, it is not surprising that in a well-designed

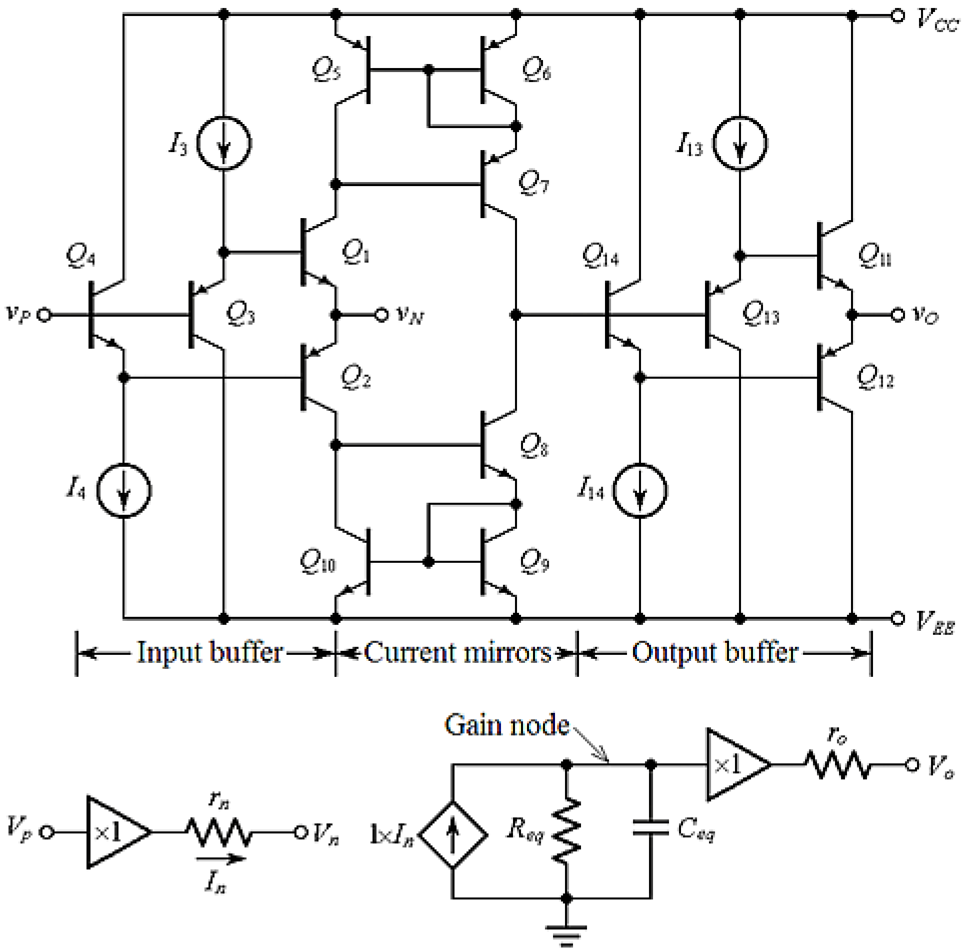

Figure 6 Current-feedback amplifier (CFA) schematic (top) and its ac equivalent (bottom).

CFA, R_{eq} is large (typically in the MΩ range). Moreover, C_{eq} is in the pF range. If we define

$$z = \left[R_{eq} // \left(1/j\omega C_{eq} \right) \right] = \frac{R_{eq}}{1 + j\omega R_{eq} C_{eq}} \tag{11}$$

then it is apparent that the gain-node voltage is zxI_n. This suggests coalescing the 1xI_n source, the gain-node elements, and the output buffer into a single current-controlled voltage source of value zI_n and output resistance r_o, in the manner depicted in **Figure 7**. (Here, two external resistances R_G and R_F have also been added to configure the CFA for closed-loop operation, to be investigated next.)

The circuit of **Figure 7** is similar to a non-inverting amplifier implemented with a conventional op amp, the main difference being that in a well-designed CFA, r_n is very small (in the range of tens of Ohms or less), whereas the input resistance of an op amp is very large (in the range of Mega-Ohms or more). Regardless, we observe that in the limit $z \to \infty$ we get $V_n \to V_p = V_i$, so we can write

$$A_{ideal} = V_o/V_i = 1 + R_F/R_G \tag{12}$$

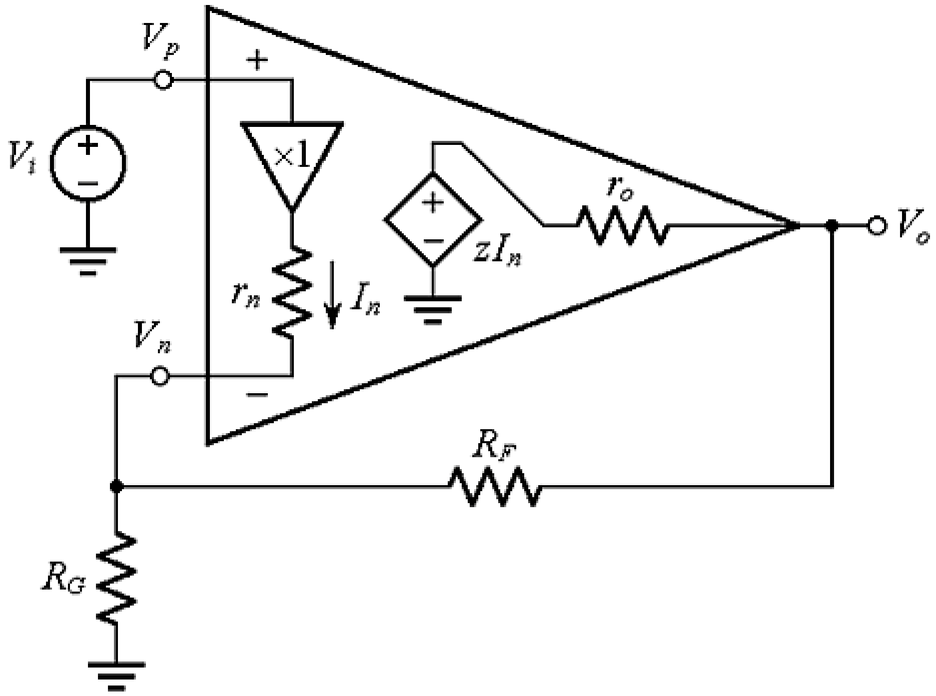

Figure 7 Configuring a CFA for non-inverting amplification. Assume the CFA has r_n = 25 Ω, r_o = 50 Ω, R_{eq} = 500 kΩ, and C_{eq} = 1.2732 pF. Moreover, let R_F = 1.25 kΩ and R_G = R_F/9.

The circuit appears to be of the *series-shunt* type, so let us attempt **TP analysis** and split it into two separate blocks: (*a*) the basic amplifier, but *modified* so as to take into account loading by the feedback network, and (*b*) the *unloaded* feedback network itself. This is shown in **Figure 8**, where the *current-controlled* voltage source zI_n has been replaced with a *voltage-controlled* voltage source $zI_n = z(V_{pn}/r_n)$ to make the CFA look even closer to the more familiar op amp. By inspection, we write

$$V_o = \frac{R_G + R_F}{r_o + R_G + R_F}\frac{z}{r_n}V_{pn} = \frac{1}{1 + r_o/(R_G + R_F)}\frac{z}{r_n}\frac{r_n}{r_n + (R_G // R_F)}V_i$$

so the modified gain is:

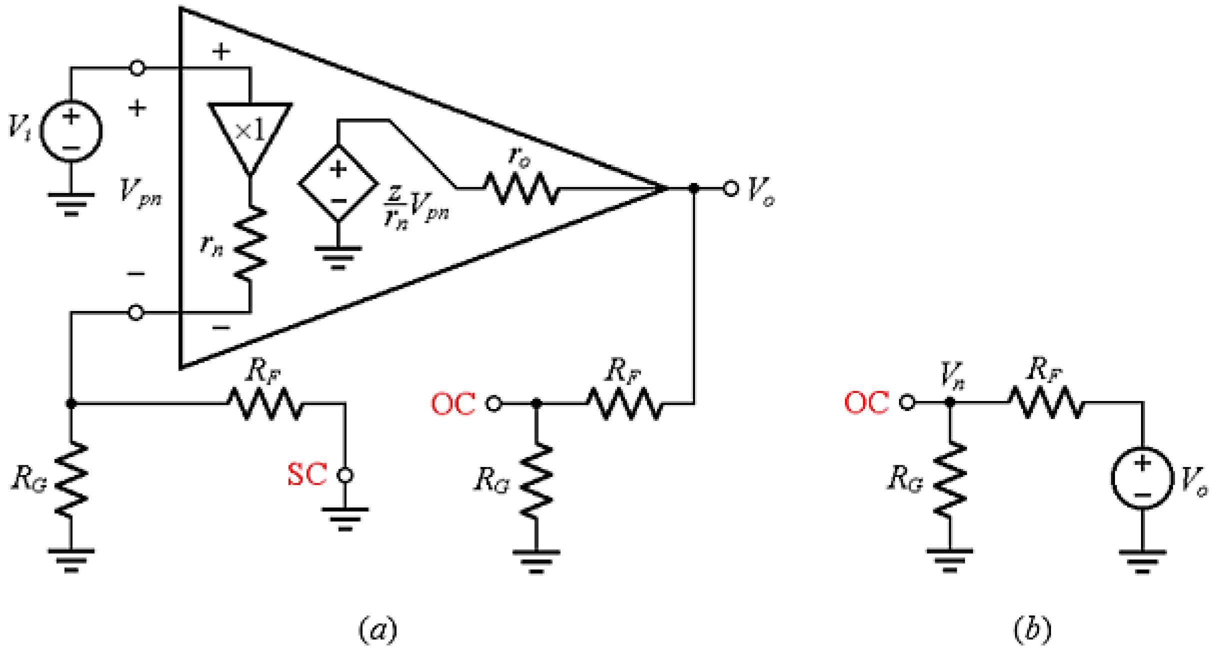

Figure 8 To perform TP analysis of the feedback circuit of Figure 7, decompose it into a modified amplifier to account for *loading* by the feedback network, and the *unloaded* feedback network.

$$a_{mod} = \frac{V_o}{V_i} = \frac{1}{1 + r_o/(R_G + R_F)} \times \frac{z}{r_n + (R_G // R_F)} \qquad (13)$$

Using the data of **Figure 7** to calculate A_{ideal} and a_{mod} and then plugging into Equation (2) gives

$$A_{TP} = \frac{v_o}{v_i} = \frac{10 \text{ V/V}}{1 + (1.554 \text{ k}\Omega)/z} \qquad (14)$$

It was shown in a previous blog [4] that for $r_n = r_o = 0$, Equation (2) would give $A = (10 \text{ V/V})/(1 + R_F/z)$, so according to TP analysis, the presence of r_n and r_o has the effect of raising R_F from its actual value of 1.25 kΩ to an apparent value of 1.554 kΩ.

We now wish to use **RR analysis**, which we know to give *exact* results, to see how good an approximation TP analysis offers. With reference to **Figure 9*a*** we have

$$V_r = zI_n = z\left(\frac{0 - V_n}{r_n}\right) = -\frac{z}{r_n}\frac{r_n // R_G}{(r_n // R_G) + R_F + r_o}V_t$$

so, the return ratio of the zI_n source is, after some algebra,

$$T = -\frac{V_r}{V_t} = \frac{z}{r_n + (1 + r_n / R_G) \times (R_F + r_o)} \tag{15a}$$

In **Figure 9*b*** we proceed in the usual manner and find

$$a_{ft} = \frac{V_o}{V_i} = \frac{1}{1 + R_F / r_o} \frac{1}{1 + r_n / \left[R_G // (R_F + r_o) \right]} \tag{15b}$$

Using the data of **Figure 7** to calculate A_{ideal}, T, and a_{ft} and then plugging into Equation (8), we get

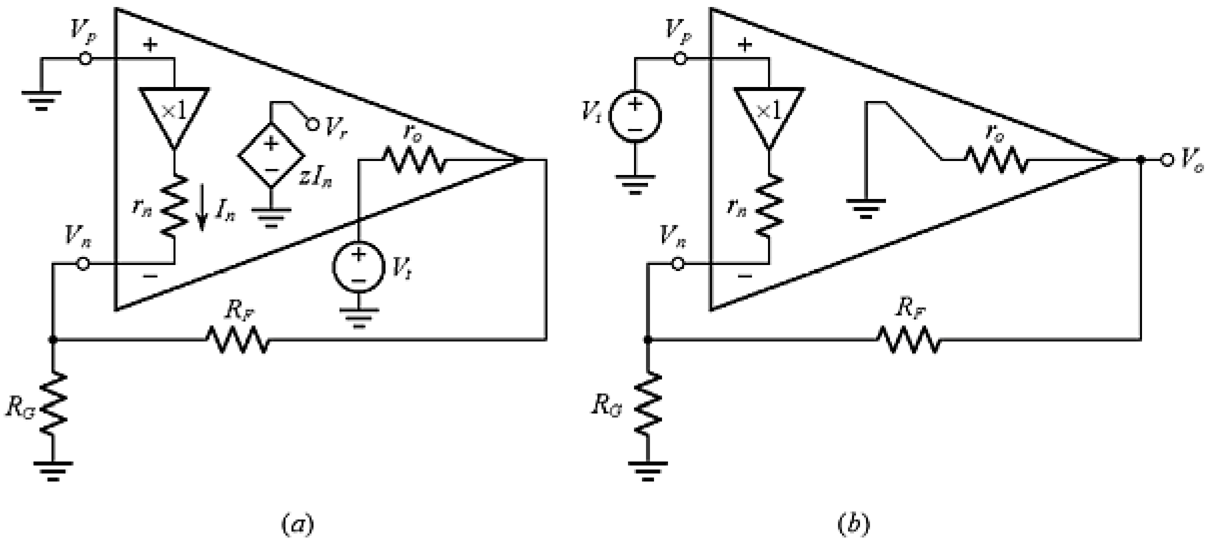

Figure 9 Using RR analysis to find (*a*) the return-ratio T of the source zI_n, and (*b*) the feedthrough gain a_{ft} around the same source.

$$A_{RR} = \frac{v_o}{v_i} = \frac{10 \text{ V/V}}{1 + (1.559 \text{ k}\Omega)/z} + \frac{(1/61.16) \text{ V/V}}{1 + z/(1.559 \text{ k}\Omega)} \tag{16}$$

Note that the apparent value of R_F is now 1.559 kΩ instead of 1.554 kΩ as provided by TP analysis. This difference is quite small in the present example, but it is still indicative of the fact that TP analysis is only approximate (whereas RR analysis is exact). The difference becomes more apparent if we consider the high-frequency asymptotic values of the gains, obtained by letting $z \to 0$ in Equations. (14) and (16),

$$\lim_{\omega\to\infty} A_{RR}(j\omega) = 1/61.16 \text{ V/V} \quad (17a)$$

$$\lim_{\omega\to\infty} A_{TP}(j\omega) = 0 \quad (17b)$$

Using PSpice with the component values of **Figure 7**, we generate the snapshot of **Figure 10** for a 1-V input. Note that in order to sustain a 10.328-V output, the dependent source needs a control current I_N of (10.328 V)/(500 kΩ) = 20.66 uA. With $I_N \neq 0$, R_F and R_G draw *different* currents, and this invalidates the unloaded feedback network postulated by TP analysis in **Figure 8*b***. Indeed, TP analysis uses *fictitious circuits* designed to facilitate the estimation of the loop gain by hand calculation. But, when performing lab measurements, we face the circuit in its *real* form of **Figure 10**, not in the *fictional* form of **Figure 8**!

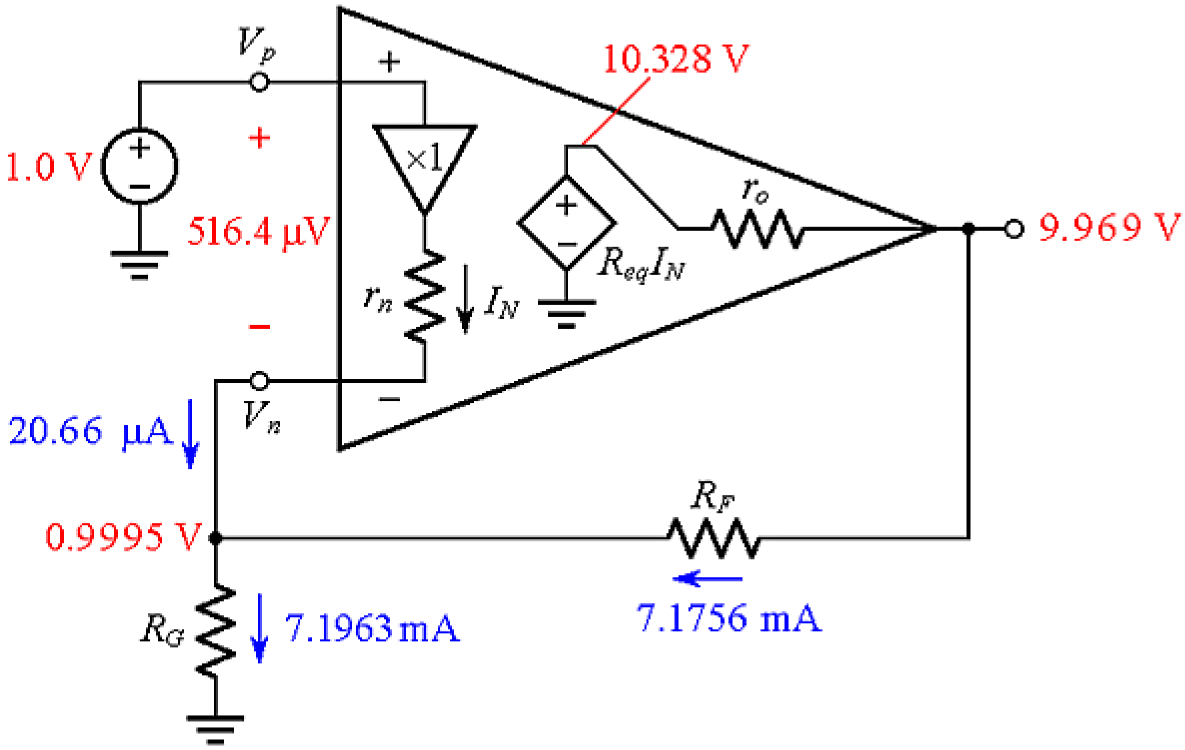

Figure 10 Voltages and currents in response to a 1.0-V dc input.

Voltage or current feedback?

Voltage or current feedback?

What about the nature of the signal being fed back to CFA's inverting input? TP analysis stipulates *voltage feedback* for *series summing* at the input, but is this the *actual* case? To find the answer, we use the *double-injection technique* pioneered by R. D. Middlebrook [5], which is suited both to SPICE simulation and to testing in the lab. This technique requires that, after putting the circuit in its dormant state, we break the feedback loop at the point of interest, and separately inject a series test-voltage V_t and a shunt test-current I_t. These stimuli will cause the circuit to react with the *forward* responses V_f and I_f, in turn accompanied by the *return* responses V_r and I_r. If we let

$$T_v = -\frac{V_r}{V_f} \text{ and } T_i = -\frac{I_r}{I_f} \tag{18}$$

then the loop gain T is obtained by exploiting the condition [5]

$$\frac{1}{1+T} = \frac{1}{1+T_v} + \frac{1}{1+T_i} \tag{19}$$

We want to know what is being fed back to the input, so we break the loop for the test-signal injections right where the feedback network meets the output node of the input buffer, as depicted in **Figure 11**. The resulting plot of **Figure 12** indicates that the makeup of T comprises both a current and a voltage component, but that the *current component prevails* since T is much *closer* to T_i than to T_v. In fact, using PSpice's cursor we find dc values of T_{i0} = 384.8, T_{v0} = 1935.5, and T_0 = 320.7, which indeed satisfy Equation (19).

We wonder what determines the mix of current and voltage feedback at a given point along the feedback path. The answer is provided by the fact that T_i and T_v satisfy the condition [5, 6]

$$\frac{1+T_i}{1+T_v} = \frac{Z_f}{Z_r} \tag{20}$$

where Z_f and Z_r are, respectively, the impedances seen looking in the *forward* and in the *return* directions from the point of test-signal injection. In **Figure 11** have $Z_f = r_n$ = 25 Ω and $Z_r = R_G//(R_F + r_o)$ = 125.5 Ω, thus confirming that at dc we have (1 + 384.8)/(1 + 1935.5) ≈ 25/125.5. It is apparent that so long as the condition

$$r_n << R_G//(R_F + r_o) \tag{21}$$

is met, *current feedback* will be predominant. In the limit $r_n \to 0$, we would have $V_{pn} \to 0$, and feedback would be *exclusively* of the current type. *None* of this is accounted for by TP analysis, which predicates exclusively voltage feedback for its fictitious circuit, regardless of whether Equation (21) is met or not. Well-designed CFA circuits generally meet Equation (21) abundantly. In fact, some CFAs use local feedback around the input buffer to achieve truly low r_n values. It is apparent that the terminology "current feedback" is quite appropriate for these devices.

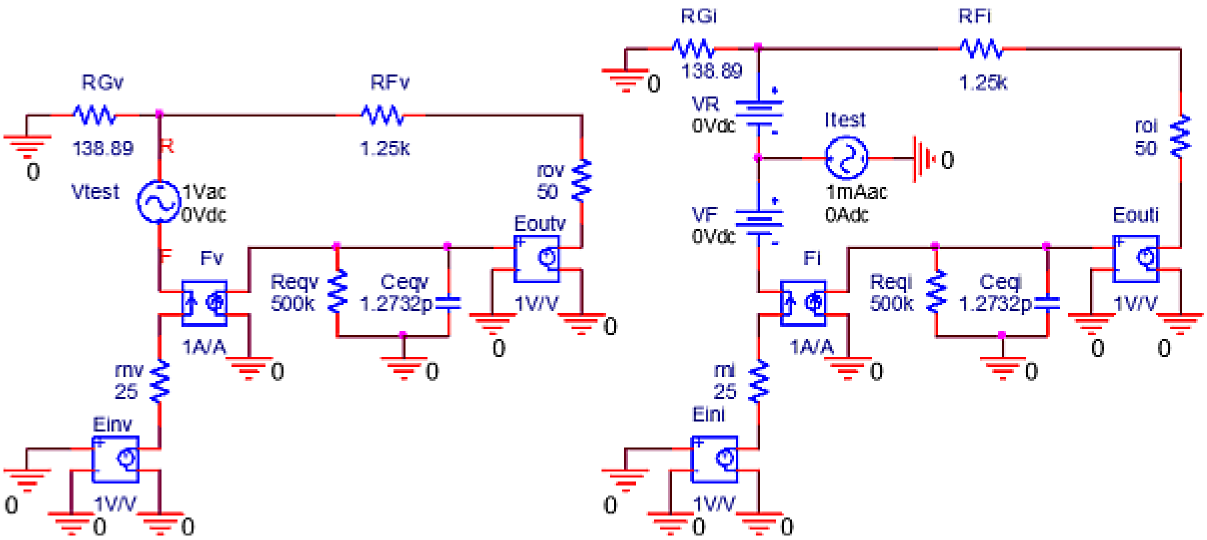

Figure 11 Using voltage and current injections to plot the loop gain T of the circuit of Figure 7.

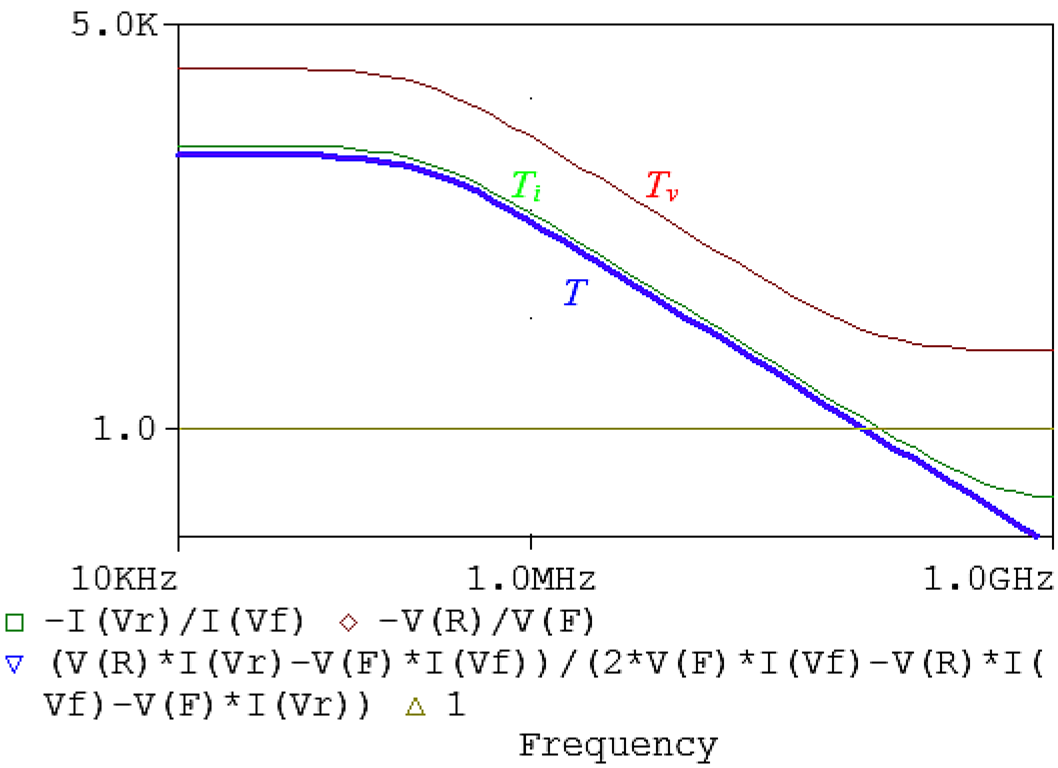

Figure 12 Frequency plots of T_v, T_i, and T.

Dual arguments can be made for circuits using conventional op amps. Shown in **Figure 13** is the popular *I-V* converter, also known as *transimpedance amplifier*. From the viewpoint of TP analysis, this is a *shunt-shunt* configuration, implying feedback of the *current* type [1]. But, is this actually the case? A well-designed *I-V* converter uses $R_F << r_i$, so if we break the circuit right at the inverting input pin to apply the double injection there, we encounter the condition $Z_f >> Z_r$ ($Z_f = r_i$, $Z_r = R_F + r_o$). By Equations (20) and (19), this implies a predominance of voltage-type feedback. In the limit $r_i \to \infty$, feedback would be *exclusively* of the voltage type; hence, the reason why conventional op amps are also called voltage-feedback amplifiers (VFAs) to distinguish them from CFAs. If, on the other hand, we implement the *I-V* converter of **Figure 13** with a CFA, for which the condition $Z_f << Z_r$ holds at the input, then feedback is indeed of the current type, and the designation shunt-shunt is appropriate in this case.

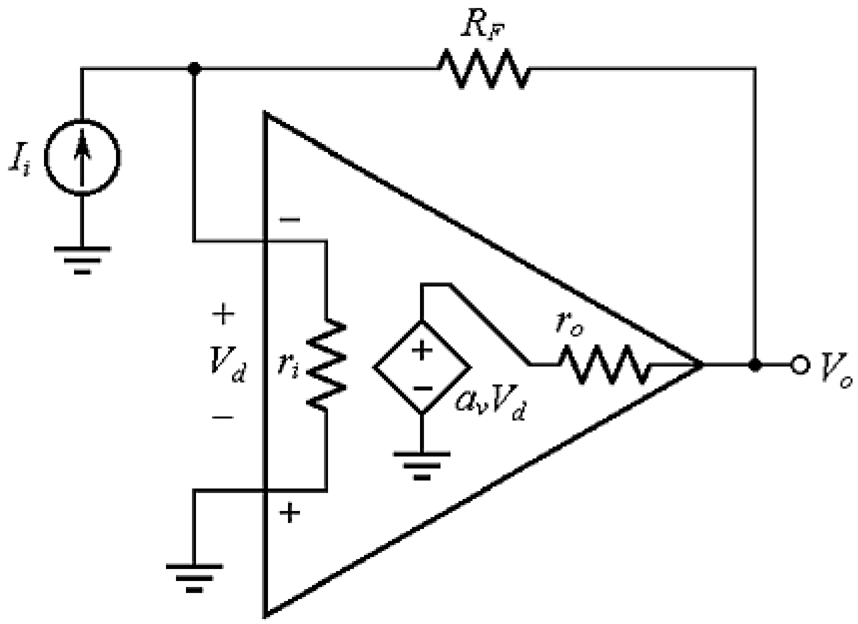

Figure 13 The shunt-shunt configuration using a conventional op amp: current or voltage feedback?

Closure

TP analysis splits a negative-feedback circuit into two *fictitious* sub-circuits for the purpose of facilitating the *paper-and-pencil calculation* of the loop gain: (*a*) a *modified* amplifier to incorporate loading by the feedback network, and (*b*) an *unloaded* feedback network. The modified circuit may no longer reflect the exact physical operation of the original circuit, and the results provided by TP analysis may not be exact either. TP analysis may also err in postulating the type of signal being actually fed back to the input.

In a well-designed VFA circuit, feedback at the input is predominantly of the voltage type, even in configurations that TP analysis would deem of the shunt-type at the input, such as the *I-V* converter of **Figure 13**. In the limit $r_i \to \infty$, the test current that we inject to measure T_i would have nowhere to flow except in the return direction, thus implying $T_i \to -I_r/0 = \infty$ and therefore $T \to T_v$, by Equation (19).

Conversely, in a well-designed CFA circuit, feedback at the input is predominantly of the current type, even in configurations that TP analysis would deem of the series-type at the input, such as the non-inverting amplifier of **Figure 7**. In the limit $r_n \to 0$, the test voltage that we inject to measure T_v would have nowhere to extend except in the return direction, thus implying $T_v \to -V_r/0 = \infty$ and therefore $T \to T_i$, by Equation. (19).

In spite of its limitations, TP analysis must be given credit for its ability to produce generally good loop-gain estimates, at least so long as feedthrough can be ignored. However, using TP analysis to question or even to attempt to invalidate the *exact* results provided by RR analysis or the double-injection technique, is a losing proposition. Mercifully, all attempts at discrediting the CFA are based on TP analysis.

Sergio Franco is an author and emeritus university professor.

References

[1] Two-port vs. return-ratio analysis

[2] Analog Circuit Design: Discrete and Integrated by Sergio Franco

[3] Quest for the Ideal Transistor?

[4] In Defense of the Current-Feedback Amplifier

[5] R. D. Middlebrook, “Measurement of Loop Gain in Feedback Systems,” *Int. J. Electronics*, Vol. 38, no. 4, pp. 485-512, 1975.

[6] Loop Gain Measurements