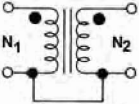
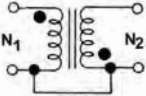
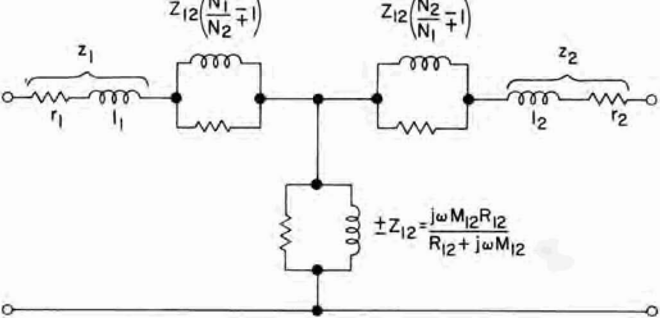


NON-INVERTING CONNECTION
Use Upper Signs



INVERTING CONNECTION
Use Lower Signs



the arms of the T network to produce the correct values of the transformer terminal impedances. That is, z_1 and z_2 are defined by

$$z_1 = Z_{11} - \left(\frac{N_1}{N_2}\right)Z_{12} \text{ and } z_2 = Z_{22} - \left(\frac{N_2}{N_1}\right)Z_{12} \qquad (1)$$

where Z_{11} and Z_{22} are the open-circuit impedances at the terminals. The components of z_1 and z_2 have physical significance if N_1 and N_2 are the actual numbers of turns (which may differ from the nominal or design values). The resistances, r_1 and r_2, are the respective resistances of the windings themselves, and are very nearly equal to the DC resistance of the wire. (At high frequency their values are increased somewhat by skin and proximity effects.) The inductances, l_1 and l_2, are the respective leakage inductances, due to flux that cuts one winding and not the other. These impedances are linear, and are relatively frequency dependent.

The equivalent circuit of Fig. 2 does not show distributed capacitance between windings, or between the windings and the core or any shields that may be used. The effect of distributed capacitance can be very difficult to calculate but, to a first approximation, it can be represented by lumped capacitances between the terminals. Alternatively, these capacitances can be regarded as changing the effective values of z_1, z_2, and Z_{12}, so that the equivalent circuit can be used.

In Fig. 1a, loading capacitance can improve the ratio accuracy of the circuit near the resonant frequency, but above this frequency the accuracy is degraded quickly. In balanced circuits (Figs. 1b and 1c) the capacitance balance, which is important, can be improved by means of external trimming capacitors.

Errors in Simple Transformers

Using the equivalent circuit of Fig. 2, we can easily calculate the ratios of interest in the circuits of Fig. 1. For the first circuit (Fig. 1a)

$$\frac{E_2}{E_1} = \frac{\dfrac{N_2}{N_1}}{1 + \dfrac{N_2 z_1}{N_1 Z_{12}}}$$

$$\approx \frac{N_2}{N_1}\left[1 - \frac{N_2}{N_1}\left(\frac{r_1}{R_{12}} + \frac{l_1}{M_{12}} + \frac{r_1}{j\omega M_{12}} + \frac{j\omega l_1}{R_{12}}\right)\right] \qquad (2)$$

Note that in Fig. 2, the set of signs to be used depends upon the connection. The lower set of signs corresponds to a voltage inversion, and would add a minus sign to equation (2). If z_1/Z_{12} were large, the voltage ratio would be imprecise, and would vary widely as Z_{12} varied with level, frequency, etc. However, by means of proper design, these error terms can be made very small so that, despite large changes in Z_{12}, the ratio remains precise.

M_{12} is proportional to N^2, while l_1 and l_2 increase less rapidly as turns are added, and are approximately proportional to N if a twisted pair of wires is used. Likewise, R_{12} is proportional to N^2, while r_1 and r_2 are proportional to N for a given wire size. Therefore, many turns of heavy wire produce a good ratio. M_{12} is also proportional to the permeability of the core—and cores with values of μ as great as 100 000 or more are not uncommon. Since R_{12} (parallel) is increased as core losses are reduced, very thin iron is used to reduce eddy-current losses.

Because of the symmetry of their magnetic fields, toroidal cores are usually used in precision transformers. While accurate ratios are possible with cores of other shapes, flux tends to cut across sharp corners, thereby failing to link some wires, which results in increased leakage inductance. Moreover, the relatively large "window" area of a toroid affords a large space for the windings, without placing them too far from the core.

The circuit of Fig. 1a, corresponding to equation (2), produces the poorest ratio accuracy of the three because one winding carries current and the other doesn't. The error is simply a result of the voltage drop, Iz_1. Using twisted windings, 1:1 ratios within about 100 ppm are common. Ratios other than 1:1 produce greater errors, due to higher leakage inductance caused by poorer winding geometry.

In the divider circuit of Fig. 1b, the mutual impedance becomes negative, as shown in the equivalent circuit of Fig. 3. For this connection

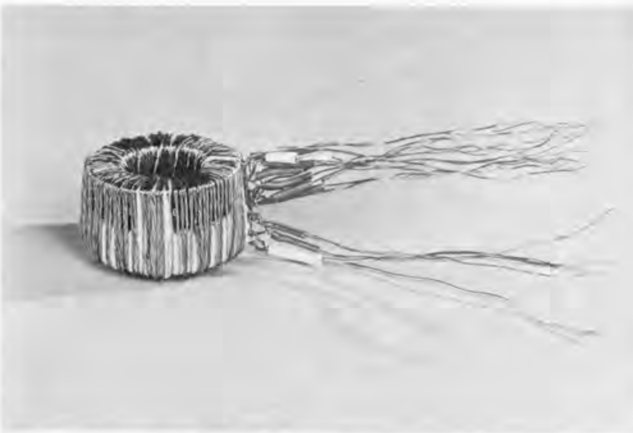
$$\frac{E_1}{E_2} = \frac{N_1}{N_2}\left[\frac{1 + \dfrac{z_1 N_2}{Z_{12}(N_1+N_2)}}{1 + \dfrac{z_2 N_1}{Z_{12}(N_1+N_2)}}\right]$$

$$\approx \frac{N_1}{N_2}\left[1 + \left(\frac{N_1 N_2}{Z_{12}(N_1+N_2)}\right)\left(\frac{z_1}{N_1} - \frac{z_2}{N_2}\right)\right] \qquad (3)$$

$$\frac{E_2}{E_s} = \frac{N_2}{N_1+N_2}\left[\frac{1 + \dfrac{z_2 N_1}{Z_{12}(N_1+N_2)}}{1 + \dfrac{(z_1+z_2)N_1 N_2}{Z_{12}(N_1+N_2)^2}}\right]$$

$$\approx \frac{N_2}{N_1+N_2}\left[1 + \left(\frac{N_1 N_2}{Z_{12}(N_1+N_2)}\right)\left(\frac{z_2}{N_2} - \frac{z_1+z_2}{N_1+N_2}\right)\right] \qquad (4)$$

Here all of the error terms include winding impedance *differences*, so that with careful design, the ratios can be made much better than those of the unsymmetrical circuit of Fig. 1a. At low frequencies the winding *resistance* becomes important. (At DC, the circuit would become simply a resistance divider.) Equations (3) and (4) indicate that the resistance per turn should be equal for both halves of the divider; therefore, both windings should use the same size wire, regardless of the number of turns. Wire from the same spool is often specified, to ensure uniformity of wire size. A good leakage-inductance balance is obtained easily for 1:1 transformers if a twisted pair is used. For other ratios, a bundle of wires is wound. For example, a 10:1 ratio would use a bundle of eleven wires, with ten of them connected in



• Experimental model of a multiple-ratio, two-stage transformer, wound on two 2½-inch-diameter cores. Referring to Fig. 4, which shows a schematic representation of this transformer, winding **a** is on the bottom core only. The other two windings consist of a twisted pair that is wound on both cores simultaneously. One wire of the pair comprises windings **b** and **d**, and the other, windings **c** and **f**. If winding **a** is disconnected, the unit becomes a simple transformer (wound on two cores) in which the in-phase 1:1 ratio error at a frequency of 1 kHz and a level of 25 V was measured as 80 ppm. With winding **a** connected, which makes the unit a two-stage transformer, the corresponding in-phase 1:1 ratio error becomes approximately 0.05 ppm.

and $$Z_{33} = z_{31} + \frac{N_3 Z_{13}}{N_1}$$

or $$Z_{33} = z_{32} + \frac{N_3 Z_{23}}{N_2} \tag{10}$$

Equations (8), (9), and (10) are useful in reducing network equations to a more meaningful form.* Together with the preceding equations, they present a complete picture of a three-winding transformer. The extension of this system to transformers with even more windings is obvious, but one should be warned that the calculations can become very tedious.

The Brooks and Holtz Two-Stage Transformer [9, 2]

Figure 4 shows a circuit that provides an accurate isolated voltage ratio (cf Fig. 1a). It was originally used to achieve an accurate current ratio, for use in a more accurate wattmeter. However, by reciprocity, if it can provide a good current ratio, then it can also provide a good voltage ratio.

In Fig. 4, the circuit is shown as a voltage-ratio device. One way to consider it is as a feedback network. The full input voltage is applied to winding *a*. As a result, a voltage appears across winding *c* that is slightly smaller than N_2E_1/N_1 by the error factor of equation (2). This error results from the voltage drop across the winding resistance and leakage inductance, caused by the magnetizing current. The open-circuit voltage in winding *b* would also be low, due to this same type of error, but it would be slightly different if the coupling between windings *a* and *b* differed from that between windings *a* and *c* (i.e., $Z_{ab} \neq N_1 Z_{ac}/N_2$). The difference between the input voltage and the "feedback" voltage across winding *b* is applied to a second transformer, which adds a voltage proportional to this difference to the output. (Actually, winding *b* does carry some current, so that its voltage is not the true open-circuit voltage—but this current is usually very small).

Another way to look at this circuit is simply to say that winding *a* supplies the magnetizing current to establish the flux in transformer T_1, and the remaining windings become two transformers with their primaries and secondaries connected in series. Since they carry very little current, these windings introduce little error. This viewpoint makes the circuit more similar to the other two-stage circuits that will be described.

Referring to Fig. 4, the open-circuit voltage ratio, E_2/E_1, is approximately

$$\frac{E_2}{E_1} \approx \frac{N_2}{N_1}\left[1 - \left(\frac{N_2^{\,2}}{N_1^{\,2}}\right)\left(\frac{z_{ab}(z_{bc}+z_{df})}{Z_{ac}Z_{df}}\right) + \left(\frac{N_2}{N_1}\right)\left(\frac{z_{ab}-z_{ac}}{Z_{ac}}\right)\right] \tag{11}$$

The first error term is analogous to the error term of equation (2), but it is multiplied by a factor that makes it extremely small.

The second term is due to imperfect sampling—i.e., the feedback voltage on winding *b* is not necessarily produced by *exactly* the same flux that links winding *c*, because the coupling may be different. Although this term is very small, it can be larger than the first term. An experimental 10:1 transformer, using toroidal cores, exhibited a ratio error of approximately 0.1 ppm (0.01 ppm of the input).

This two-stage transformer can also be represented by the equivalent T network of Fig. 2, if the constants of Fig. 2 are replaced by:

$$z_1 \approx z_{ac} - z_{ab} + \left(\frac{N_2}{N_1}\right)\left(\frac{z_{ab}(z_{bc}+z_{df})}{Z_{df}}\right) \tag{12}$$

$$z_2 \approx z_{cb} + z_{fd} + \left(\frac{N_2}{N_1}\right)^2 (z_{ba}+z_{df}) \tag{13}$$

$$Z_{12} \approx Z_{ac} + \frac{N_2}{N_1} z_{ab} \approx Z_{ac} \tag{14}$$

Note that the effective primary-winding impedance, z_1, is very small, but the secondary-winding impedance, z_2, is increased by the transformed impedance of the primary circuit, so that the power efficiency is not improved.

This two-transformer network, as well as those yet to be discussed, can be constructed as a single, two-core device. Because windings *b* and *d* have the same number of terms, they can consist of a single winding on two cores. Similarly, windings *c* and *f* can comprise a single winding on two cores. To construct this transformer, winding *a* would be placed on one core, a second core added, and then the other windings placed on both cores.

The Two-Stage Divider [10]

Returning to Fig. 1b, the divider errors are caused by unequal voltage drops across the winding impedances. These errors could be reduced if the current could be reduced. The circuit shown in Fig. 5 might be tried for this purpose, with the intent that the added winding supply the magnetizing current. Although winding *a* does supply some magnetizing

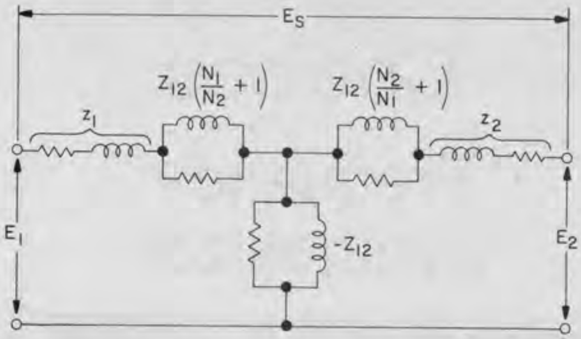


Fig. 3. Equivalent T network for the transformer shown in Fig. 1b. In this equivalent circuit, the mutual impedance is negative.

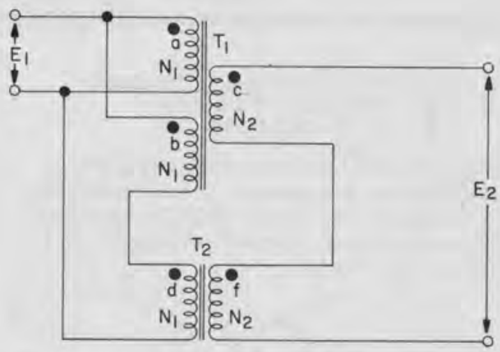


Fig. 4. Schematic representation of a two-stage transformer, that provides the means for obtaining an accurate voltage ratio, and isolation as well.

*From these equations one can derive the interesting and sometimes useful relationship:

$$\frac{z_{12}-z_{13}}{N_1^{\,2}} + \frac{z_{23}-z_{21}}{N_2^{\,2}} + \frac{z_{31}-z_{32}}{N_3^{\,2}} = 0$$

current, nonetheless the total current would divide between the two paths, depending upon the winding impedances, and the improvement would not be substantial. However, if large impedances of the proper ratio were added in series with windings *b* and *c*, thereby limiting the current in these windings to a small value, then winding *a* would have to supply most of the magnetizing current.

The most accurate way to add impedances of the correct ratio is by means of a second transformer divider with the *same* turns ratio, as shown in Fig. 6. Although this adds additional winding impedances that would, in themselves, tend to degrade the ratio, the current reduction is so substantial that the effect of winding-impedance unbalance can be extremely small.

The calculated approximate ratios for this device are

$$\frac{E_1}{E_2} \approx \frac{N_1}{N_2}\left[1 + \frac{z_{ac}-z_{ab}}{Z_{aa}} + \left(\frac{N_1N_2(N_1z_{ab}+N_2z_{ac})}{(N_1+N_2)^2Z_{aa}Z_{df}}\right)\left(\frac{z_{bc}+z_{df}}{N_1} - \frac{z_{cb}+z_{fd}}{N_2}\right)\right] \quad (15)$$

$$\frac{E_2}{E_s} \approx \frac{N_2}{N_1+N_2}\left[1 + \frac{N_1z_{ab}-N_2z_{ac}}{(N_1+N_2)Z_{aa}} + \left(\frac{N_1N_2(N_1z_{ab}+N_2z_{ac})}{(N_1+N_2)^2Z_{aa}Z_{df}}\right)\left(\frac{z_{cb}+z_{fd}}{N_2} - \frac{z_{bc}+z_{df}+z_{cb}+z_{fd}}{N_1+N_2}\right)\right] \quad (16)$$

Equations (15) and (16) should be compared with equations (3) and (4). The winding-impedance error terms are multiplied by small factors, as one would expect. However, note the new error terms

$$[(z_{ac}-z_{ab})/Z_{aa} \text{ and } (N_1z_{ab}-N_2z_{ac})/(N_1+N_2)Z_{aa}]$$

that result from unbalanced coupling to winding *a*. While these errors can be extremely small, they can, nonetheless, be just as important as the other errors, which can be even smaller. The better the symmetry of field and windings, the smaller this coupling error will be.

The Gibbings Transformer [11]

The circuits of Figs. 1b and 1c are similar, except that an extra winding is added in 1c to supply the drive. Similarly, adding such a winding to the circuit of Fig. 6 results in the Gibbings circuit, shown in Fig. 7. In the circuit of Fig. 1c, the open-circuit voltage ratio depends only upon a mutual-impedance ~~inductance~~ ratio, and a two-stage circuit cannot improve on that. However, in bridge circuits it places winding impedances in series with the impedances being compared, so that it is unsuitable for low-impedance measurements.

In the bridge circuit of Fig. 1c, the approximate impedance ratio at null is

$$\frac{Z_1}{Z_2} \approx \frac{N_1}{N_2}\left[1 + \frac{z_{32}-z_{31}}{Z_{33}} + \frac{z_{21}}{Z_2} - \frac{z_{12}}{Z_1} + \left(\frac{z_{31}-z_{32}}{Z_{33}}\right)\left(\frac{(N_1+N_2)^2 Z_{33}}{N_3{}^2(Z_1+Z_2)}\right)\right] \quad (17)$$

Here we have three types of errors, the first of which is the open-circuit error due to unequal coupling.

$$\frac{Z_{31}}{Z_{32}} \approx \frac{N_1}{N_2}\left[1 + \frac{z_{32}-z_{31}}{Z_{33}}\right] \quad (18)$$

The next two terms take into account the impedance in series with Z_1 and Z_2.

If $N_1/N_2 = (Z_1+z_{12})/(Z_2+z_{21})$, then

$$\frac{Z_1}{Z_2} \approx \frac{N_1}{N_2}\left[1 + \frac{z_{21}}{Z_2} - \frac{z_{12}}{Z_1}\right] \quad (19)$$

The last term is the open-circuit error multiplied by a loading factor. It is caused by the increased effect of the primary-winding impedance as the circuit is loaded and the input current increases.

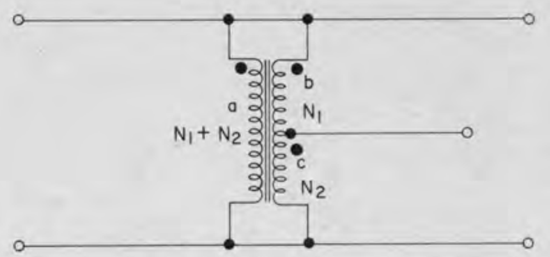


Fig. 5. Three-winding voltage divider that provides little advantage over the circuit of Fig. 1b. It could be improved by adding large impedances of the appropriate ratio in series with windings **b** and **c**.

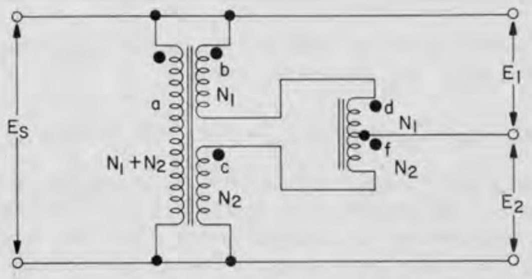


Fig. 6. The addition of a second transformer to the circuit of Fig. 5 facilitates accurate introduction of impedances of the correct ratio. The effect of the added winding impedance is far outweighed by the reduction of current in the **b** and **c** windings.

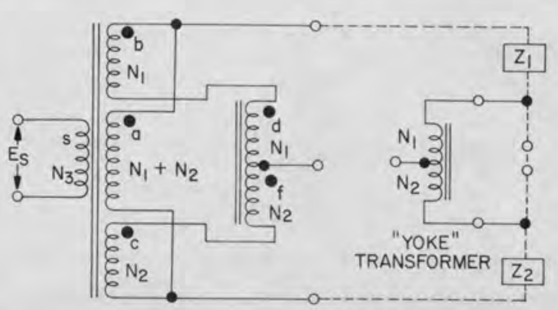


Fig. 7. The Gibbings transformer which adds a separate drive winding to the circuit of Fig. 6, is suitable for low-impedance comparisons. With the further addition of a "yoke" transformer to reduce the errors caused by the leads connecting Z_1, this circuit provides a good four-terminal bridge.