AN866

Designing Operational Amplifier Oscillator Circuits For Sensor Applications

Author: *Jim Lepkowski*
Microchip Technology Inc.

INTRODUCTION

Operational amplifier (op amp) oscillators can be used to accurately measure resistive and capacitive sensors. Oscillator design can be simplified by using the procedure discussed in this application note. The derivation of the design equations provides a method to select the passive components and determine the influence of each component on the frequency of oscillation. The procedure will be demonstrated by analyzing two state-variable RC op-amp oscillator circuits.

SENSOR APPLICATIONS

State-variable oscillators are often used in sensor conditioning applications because they have a reliable start-up and a low sensitivity to stray capacitance. The absolute and ratio state-variable oscillators can be used to accurately detect both resistive and capacitive sensors. However, this application note will only analyze capacitive applications. The state-variable's three op-amp topology provides for a more dependable oscillation start-up than a single op amp oscillator. The virtual ground voltage at the inverting terminal of the amplifiers provides for immunity from stray capacitance, which is important in sensor applications, because the sensor capacitance is often only 10 to 100 pF. In addition, the state-variable oscillator does not require matched capacitors or capacitors that have a terminal connected to ground.

The absolute oscillator provides an output frequency that is proportional to the square root of the product of two capacitors (i.e., freq. $\propto (C_1 \times C_2)^{1/2}$). Absolute quartz pressure sensors and humidity sensors are examples of capacitive sensors that can use the absolute oscillator. Also, this circuit can be used with resistive sensors, such as RTDs, to provide a temperature-to-frequency conversion.

The ratio oscillator provides an output frequency that is proportional to the square root of the ratio of two capacitors (i.e., freq. $\propto (C_4 / C_3)^{1/2}$). The ratio oscillator can be used to cancel the effect of a fluid-level sensor with a varying dielectric constant, such as an oil level sensor. An oil level sensor consists of two capacitances that are formed by tubes where the fluid serves as the dielectric media. The measurement capacitor (C_{MEAS}) is partially covered by fluid and detects the level of the oil in the tank. In contrast, the compensation sensor (C_{COMP}) is completely buried in the fluid. When the ratio C_{MEAS} / C_{COMP} is calculated, the dielectric constant of the oil is canceled. Air pressure and acceleration sensors can also use the ratio sensor to minimize the error that occurs from the variance of the dielectric constant over temperature.

TRANSDUCER SYSTEM

A block diagram of a typical sensor system is shown in Figure 1. The oscillation frequency can be found by counting the number of clock pulses (i.e., MHz) in a time window that is formed by the square wave output (i.e., kHz) of a comparator circuit. The counter and comparator circuits can be implemented with a PICmicro® microcontroller.

The PICmicro microcontroller can be used to provide curve-fit temperature correction for precision sensing applications. Temperature correction can be accomplished by implementing a curve-fitting routine with data obtained by calibrating the sensor over the operating range. The temperature correction data can be stored in the E^2 memory of the PICmicro microcontroller. A silicon IC sensor can provide the temperature of the sensor.

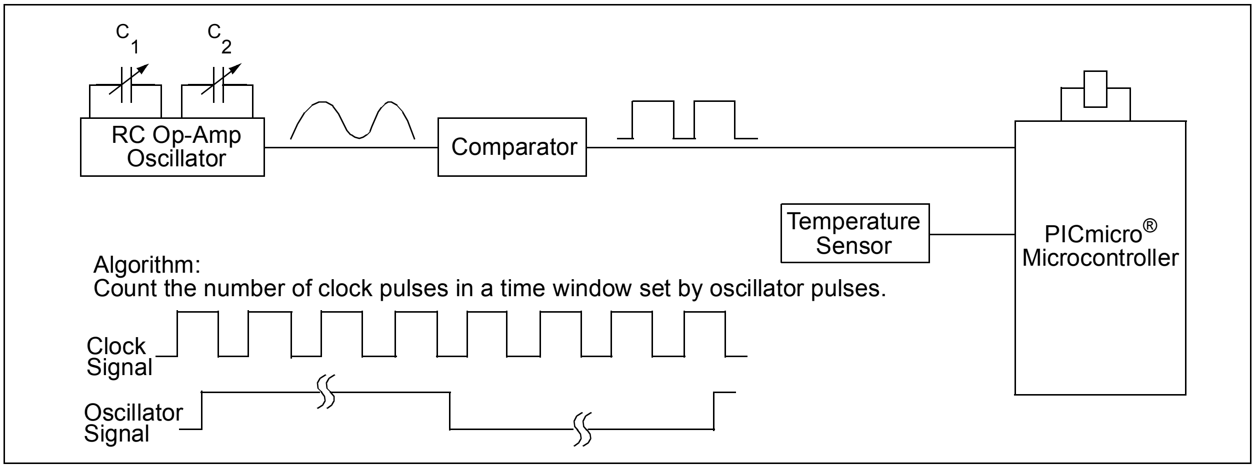


FIGURE 1: *Typical RC Operational Amplifier Oscillator Sensor System.*

OSCILLATOR THEORY

An oscillator is a positive feedback control system that generates an output without requiring an input signal. A sustained oscillation is initiated by factors such as noise pick–up or power supply transients. Figure 2 shows a block diagram of an oscillator, along with the definition of the mathematical terms that describe an oscillator.

The design equations of an oscillator are determined by analyzing the denominator of the transfer equation T(s) of the circuit. The poles of the denominator of T(s), or equivalently, the zeroes of the characteristic equation (Δs), determine the time domain behavior and stability of the system. An oscillator is on the border line between a stable and an unstable system and is formed when a pair of poles are on the imaginary axis.

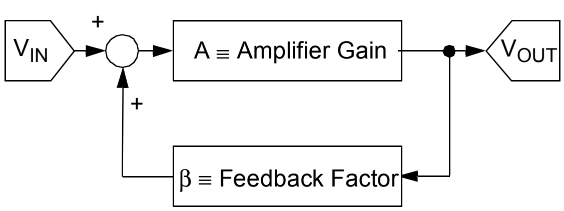
The magnitude and phase equations of an oscillator also must be analyzed. If the magnitude of the loop-gain is greater than one and the phase is zero, the amplitude of oscillation will increase exponentially until a factor in the system, such as the supply voltage, restricts the growth. In contrast, if the magnitude of the loop-gain is less than one, the amplitude of oscillation will exponentially decrease to zero.

OSCILLATOR DESIGN PROCEDURE

Listed below is a procedure to design RC operational amplifier oscillators. Refer to the design equation section of this application note for additional information on deriving oscillator design equations.

Step 1: Find LG and Δs

The oscillation frequency is determined by finding the poles of the denominator of the transfer equation T(s), or equivalently, the zeroes of the numerator N(s) of the characteristic equation (Δs). Mason's Reduction Theorem, shown in Appendix A, provides a method of obtaining Δs. Then Δs is found by breaking the feedback loop and obtaining the gain equation at each op-amp in order to calculate the loop-gain.



$$T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{A}{1 - A\beta} = \frac{A}{1 - LG} = \frac{A}{\Delta s} = \frac{A}{\frac{N(s)}{D(s)}}$$

where: A x β = LG $\equiv$ loop gain

Δs $\equiv$ characteristic equation

If V_{IN} = 0, then T(s) = ∞ when Δs = 0

FIGURE 2: *Oscillator Block Diagram.*

Step 2: Solve N(s) = 0

The second step in the procedure determines the zeroes of N(s). Routh's stability criterion, shown in Appendix B, provides a method that determines the zeroes of the characteristic equation without the necessity of factoring the equation.

First, the Routh test consists of forming a coefficient array from N(s). Next, the procedure substitutes $s = j\omega_o$ for s, with the summation of the row set to zero. If the row equation produces a non-trivial solution for ω_o, the procedure is complete and the frequency of oscillation is equal to ω_o. If the row equation does not yield an equation that can be solved for ω_o, the procedure continues with the next row in the Routh array. Usually, it is necessary only to complete the first two or three rows of the Routh array to produce an equation that can be solved for ω_o.

Step 3: Sub-Circuit Design Equations

The third step in the design procedure analyzes the sub-circuits formed at each amplifier. The sub-circuit equations are formed by obtaining the gain equation and pole/zero locations for each amplifier.

Step 4: Verify $|LG| \geq 1$

The final step in the procedure verifies that the loop-gain is equal to, or greater than, one after the R and C component values have been chosen. This step is also required to verify that the amplifiers do not saturate, which will result in an error in the oscillation frequency.

AMPLIFIER SELECTION CRITERIA

The appropriate op amp to use in a sensor oscillator is determined by the required accuracy and acceptable distortion of the oscillation frequency. The design equations assume that the amplifiers are ideal. However, op amps have a finite gain bandwidth product (GBW), a limited slew rate (SR) and full power bandwidth (f_P). The non-ideal characteristics of the amplifier will lower the oscillation frequency at high frequencies and may also result in a design with poor start-up characteristics. Note that the total harmonic distortion specification of the amplifiers is critical for oscillators that are used as sine wave references. However, the shape of the waveform is not critical in most sensor applications because only the frequency of the output is measured.

Several general design rules can be used to select an op amp for an oscillator circuit. First, the GBW should be a factor of 10 to 100 higher than the maximum oscillation frequency. Next, the full-power bandwidth, defined as $f_P = SR / (2\pi V_P)$, where V_P is the voltage swing ($V_{O(max)} - V_{O(min)}$) of the output signal, should be at least 2 times greater than the maximum oscillation frequency. For example, the MCP6024 quad amplifier has a GBW = 10 MHz (typ.), SR = 7 V/µs (typ.) and a f_P of 400 kHz, with V_{DD} = 5V. An oscillator with a maximum frequency of 100 kHz can be implemented with the MCP6024 with enough design margin that the non-ideal characteristics of the amplifier can be neglected.

ABSOLUTE STATE-VARIABLE OSCILLATOR

Circuit Description

The schematic of the absolute circuit is shown in Figure 3. The state-variable oscillator consists of two integrators and an inverter circuit. Each integrator provides a phase shift of 90°, while the inverter adds an additional 180° phase shift. The total phase shift of 360° of the feedback loop produced by the three amplifiers results in the oscillation. The first integrator stage consists of amplifier A_1, resistor R_1 and sensor capacitance C_1. The second integrator consists of amplifier A_2, resistor R_2 and sensor capacitance C_2. The inverter stage consists of amplifier A_3, resistors R_3 and R_4 and capacitor C_4. The addition of capacitor C_4 helps ensure oscillation start-up by providing an additional phase shift.

The absolute oscillator does not require a limit circuit if rail-to-rail input/output (RRIO) amplifiers are used and the gain of the inverter stage (A_3) is equal to one (i.e., $R_3 = R_4$). The sinewave output of the signal will swing within approximately 50 mV of the V_{DD} and V_{DD} power rails as shown in Figure 5.

A complementary output voltage comparator (A_4) is used to convert the oscillator's sinewave output to a square wave digital signal. The comparator functions as a zero-crossing detector and the switching point is equal to the virtual ground voltage (i.e., $V_{DD}/2$). Resistor R_9 is used to provide additional hysteresis (V_{HYS}) to the comparator. Listed below is the hysteresis equation.

EQUATION:

$$V_{HYS} = \frac{R_8}{R_8 + R_9} \times (V_{O(max)} - V_{O(min)})$$

$$V_{HYS} \cong \frac{R_8}{R_8 + R_9} \times V_{DD}$$

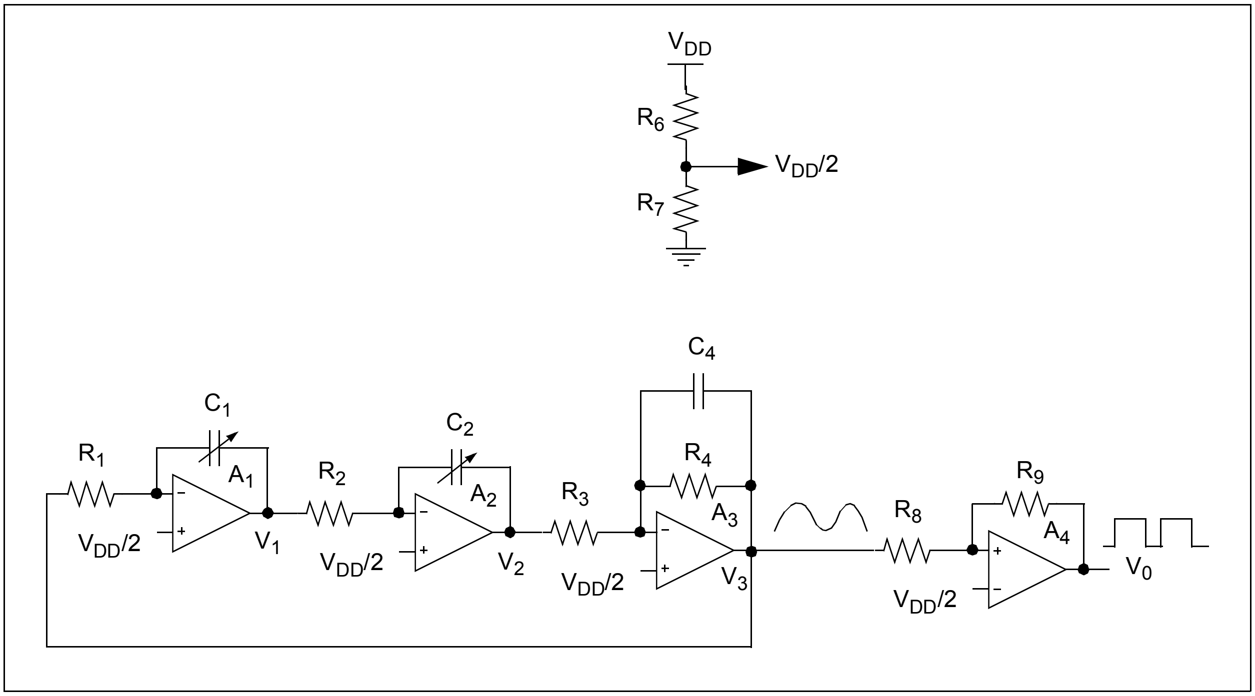


FIGURE 3: Absolute Oscillator Schematic.

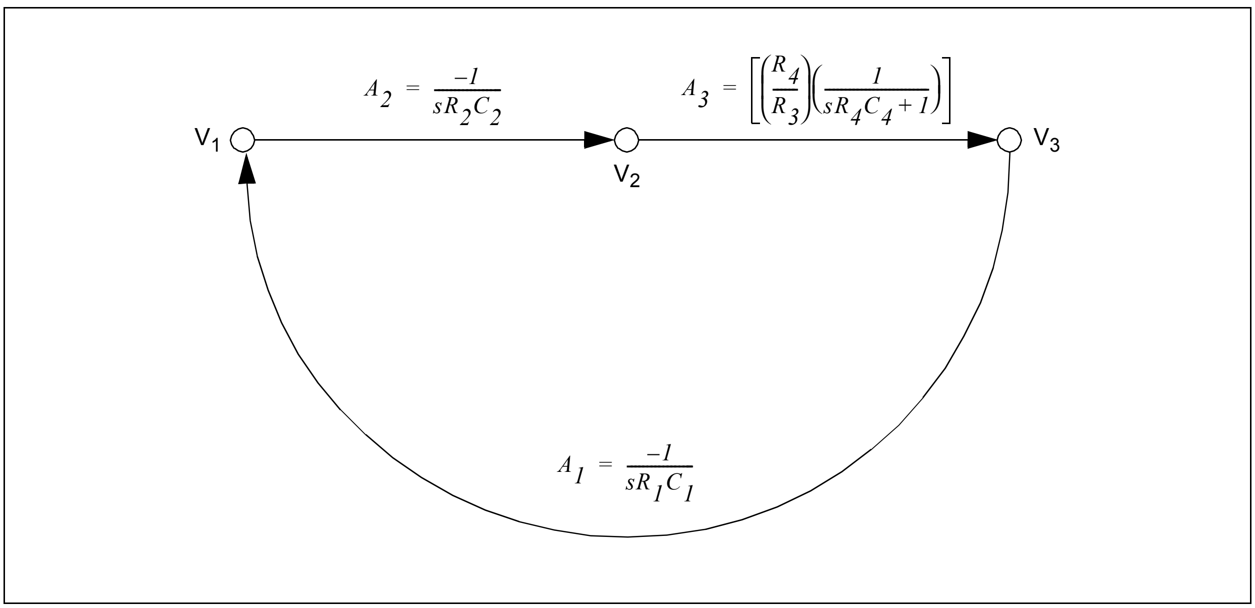


FIGURE 4: Absolute Oscillator Signal Flow Diagram.

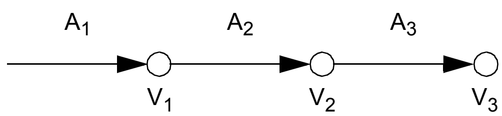
ABSOLUTE STATE-VARIABLE

Design Equations

STEP 1: FIND LG AND ΔS

$$T(s) = \frac{A}{1 - LG} = \frac{A}{\Delta s} = \frac{A}{\frac{N(s)}{D(s)}}$$

The loop-gain is found by breaking the loop in the signal flow diagram of Figure 4, as shown below.



$$\begin{aligned}
A_1 &= -1/(sR_1C_1) \\
A_2 &= -1/(sR_2C_2) \\
A_3 &= -Z_4/Z_3 \\
&= -(R_4 \parallel C_4)/R_3 \\
&= -[(R_4/R_3)(1/(sR_4C_4 + 1))]
\end{aligned}$$

$$\begin{aligned}
LG &= A_1 \times A_2 \times A_3 \\
&= [-1/(sR_1C_1)][-1/(sR_2C_2)][(-R_4/R_3)(1/(sR_4C_4 + 1))] \\
&= -\left[R_4/\left(s^3R_1R_2R_3R_4C_1C_2C_3C_4 + s^2R_1R_2R_3C_1C_2\right)\right]
\end{aligned}$$

$$\Delta s = N(s)/D(s) = 1 - LG$$

$$= 1 - [-R_4/(s^3R_1R_2R_3R_4C_1C_2C_4 + s^2R_1R_2R_3C_1C_2)]$$

$$= \frac{[s^3R_1R_2R_3R_4C_1C_2C_4 + s^2R_1R_2R_3C_1C_2 + R_4]}{[s^3R_1R_2R_3R_4C_1C_2C_4 + s^2R_1R_2R_3C_1C_2]}$$

$$N(s) = s^3R_1R_2R_3R_4C_1C_2C_4 + s^2R_1R_2R_3C_1C_2 + R_4$$

STEP 2: SOLVE N(s) = 0

The zeros of the characteristic equation are determined by using the Routh stability test.

$$\begin{aligned}
N(s) &= a_0s^3 + a_1s^2 + a_2s + a_3 \\
a_0 &= R_1R_2R_3R_4C_1C_2C_4 \\
a_1 &= R_1R_2R_3C_1C_2 \\
a_2 &= 0 \\
a_3 &= R_4
\end{aligned}$$

Routh Stability Test Coefficient Array
row s^3 $a_0 + a_2 = 0$
row s^2 $a_1 + a_3 = 0$

Row s^3 produces a trivial solution ($\omega_o = 0$):

$$a_0(j\omega_o)^3 + 0 = 0$$

The procedure continues by analyzing row s^2 to determine when the equation is equal to zero.

Let $s = j\omega_o$

$$\begin{aligned}
a_1s^2 + a_3 &= (R_1R_2R_3C_1C_2)(j\omega_o)^2 + R_4 = 0 \\
0 &= R_4 - R_1R_2R_3C_1C_2\omega_o^2 \\
\omega_o^2 &= [R_4/(R_1R_2R_3C_1C_2)] \\
\omega_o &= [R_4/(R_1R_2R_3C_1C_2)]^{1/2}
\end{aligned}$$

$$P = 2\pi/\omega_o = 2\pi/[R_4/(R_1R_2R_3C_1C_2)]^{1/2}$$

Note that C_4 does not appear in the oscillation equation. C_4 and R_4 form a low-pass filter. The gain of amplifier A_3 will not be a function of C_4 if the oscillation frequency is less than the cut-off frequency of the filter.

If:

1. $R_1 = R_2 = R$
2. $C_1 = C_2 = C$
3. $R_3 = R_4$

Then:

$$P = 2\pi RC$$

STEP 3: SUB-CIRCUIT DESIGN EQUATIONS

Integrator A_1

$$\begin{aligned}
Gain\ A_1 &= -1/(2\pi f R_1C_1) \\
Pole\ f_{p1} &= 1/(2\pi R_1C_1)
\end{aligned}$$

Integrator A_2

$$\begin{aligned}
Gain\ A_2 &= -1/(2\pi f R_2C_2) \\
Pole\ f_{p2} &= 1/(2\pi R_2C_2)
\end{aligned}$$

Integrator A_3

$$\begin{aligned}
Gain &= -[(R_4/R_3)(1/(sR_4C_4 + 1))] \\
Gain &\cong -R_4/R_3
\end{aligned}$$

STEP 4: VERIFY $|LG| \geq 1$

Assume:

1. $R_1 = R_2 = R$
2. $C_1 = C_2 = C$
3. $R_3 = R_4$

$$\begin{aligned}
|A_1| = |A_2| = |A_3| &= 1 \\
LG = |A_1 \times A_2 \times A_3| &= 1 \\
V_1 &= |A_1 \times V_3| \\
V_2 &= |A_2 \times V_1| \\
V_3 &= |A_3 \times V_2|
\end{aligned}$$

Note that a voltage limit circuit should be added if rail-to-rail input/output operational amplifiers are not used, or if the gain of the inverter is not equal to one (i.e., $R_3 \neq R_4$). A limit circuit is required to prevent the frequency error that will result from the saturation delay time of the amplifiers.

ABSOLUTE STATE-VARIABLE

Test Results

The components used in the evaluation design are listed below. Note that the capacitive sensor (i.e., C_1 and C_2) was simulated with discrete capacitors.

$R_1 = R_2 = R_6 = R_7 =$ 32.7 kΩ
$R_3 = R_4 =$ 10 kΩ
$R_8 =$ 1 kΩ
$R_9 =$ 1 MΩ
$C_1 = C_2 =$ see Table 1
$C_4 =$ 18 pF
$V_{DD} =$ 5.0V
$A_1, A_2, A_3 \equiv$ MCP6024 (quad RRIO, GBW = 10 MHZ)
$A_4 \equiv$ MCP6541Push-Pull Output Comparator

The measured and calculated oscillation frequency is shown in Table 1. Figure 5 shows the oscillation waveform when $C_1 = C_2 = 220$ pF. The error in the measured oscillation is attributed to the accuracy of the test equipment.

TABLE 1: ABSOLUTE OSCILLATOR TEST RESULTS

Capacitor Values	Calculated Oscillation Period (Frequency)	Measured Oscillation Period (Frequency)
$C_1 = C_2 = 47$ pF	9.66 µs (103.6 kHz)	10.0 µs (100.0 kHz)
$C_1 = C_2 = 56$ pF	11.5 µs (86.9 kHz)	12.0 µs (83.3 kHz)
$C_1 = C_2 = 82$ pF	16.9 µs (59.4 kHz)	16.6 µs (60.2 kHz)
$C_1 = C_2 = 100$ pF	20.6 µs (48.7 kHz)	21.0 µs (47.6 kHz)
$C_1 = C_2 = 150$ pF	30.8 µs (32.6 kHz)	30.0 µs (33.3 kHz)
$C_1 = C_2 = 220$ pF	45.2 µs (22.1 kHz)	46.0 µs (21.7 kHz)

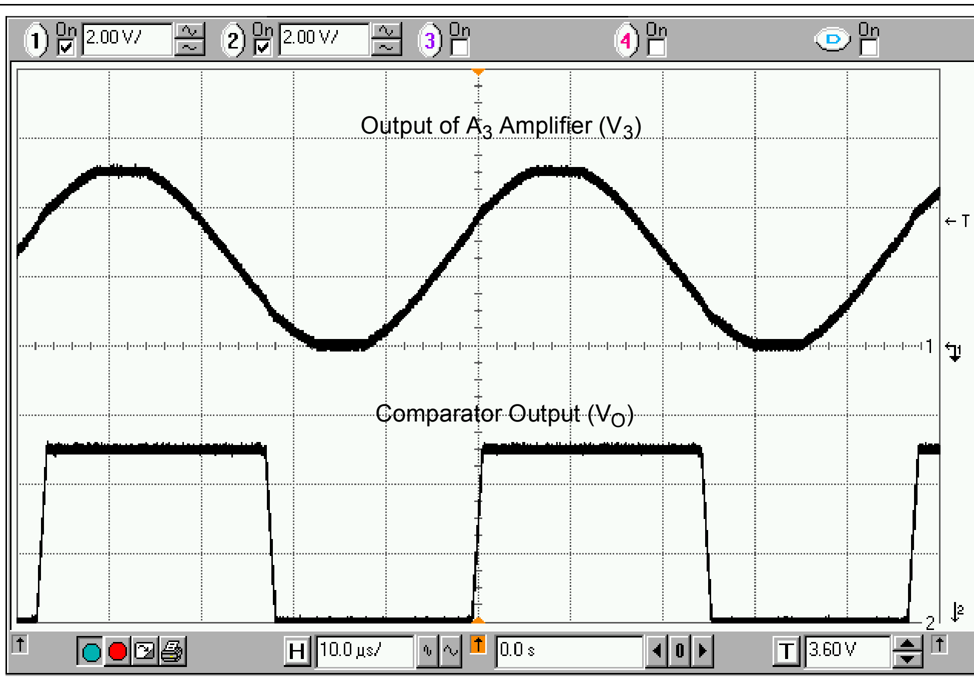


FIGURE 5: Absolute Oscillator Test Results ($C_1 = C_2 = 220$ pF).

RATIO STATE-VARIABLE OSCILLATOR

Circuit Description

The schematic of the ratio circuit is shown in Figure 6. This circuit consists of two integrators and a differentiator circuit. The integrators formed by amplifiers A_1 and A_2 are identical to the integrators used in the absolute circuit. The differentiator stage is formed by amplifier A_3, resistors R_3, R_4 and R_5, and the sensor capacitors C_3 and C_4 to provide a 180° phase shift. The comparator (A_4) used to convert the sinewave output to a square wave digital signal is identical to the absolute oscillator circuit.

A Bode plot of the differentiator stage is provided in Figure 8. The values of resistors R_3, R_4 and R_5 are selected to set the break frequencies of the differentiator stage so that the gain of the stage is equal to $-C_3/C_4$ at the oscillation frequency. Resistor R_5 is also used to provide a DC current path around capacitor C_3 in order to initiate oscillation at power-up.

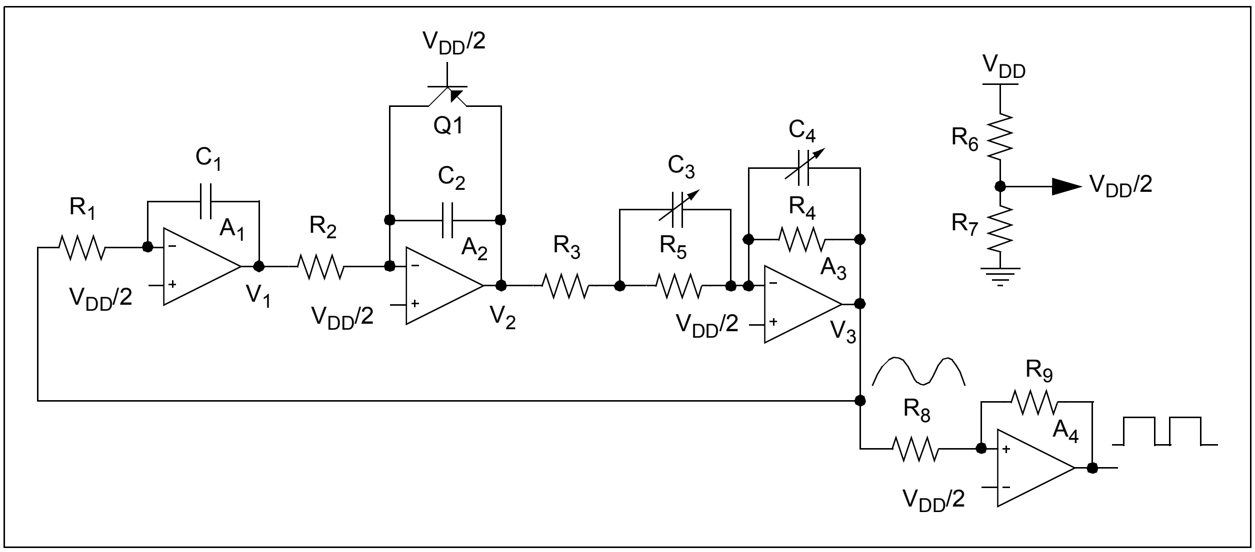


FIGURE 6: Ratio Oscillator Schematic.

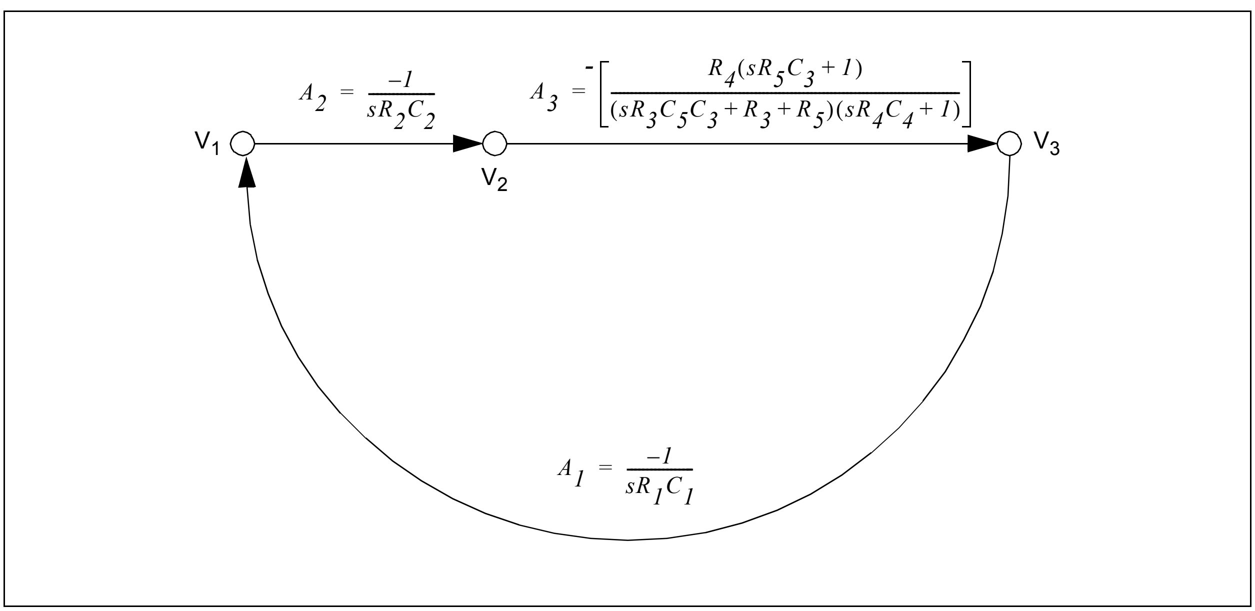


FIGURE 7: Ratio Oscillator Signal Flow Diagram.

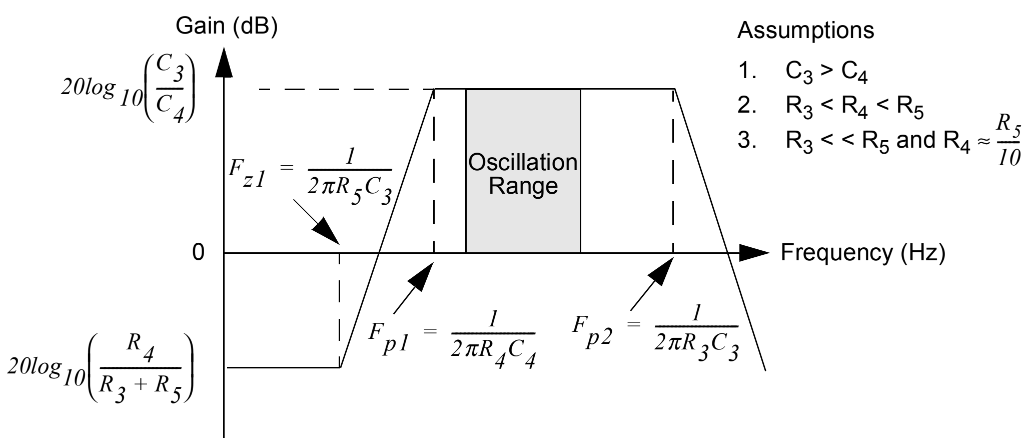


FIGURE 8: *Bode Plot of Differentiator Amplifier.*

Voltage Limit Circuit

The ratio oscillator uses a limit circuit to accommodate the varying gain requirement of the circuit. It may be necessary to add a voltage limit or clamp circuit to the oscillator to prevent the amplifiers from saturating and avoid slew rate limitations. The voltage limit circuit formed by PNP transistor Q_1 is used to create the maximum voltage limit. The clamping voltage of the limit circuit is provided below:

EQUATION:

$$V_{Max\_Limit} = V_{Q1\_base} + V_{Q1\_base\text{-}to\text{-}emitter}$$
$$V_{Max\_Limit} \cong V_{Q1\_base} + 0.7V$$

In single-supply applications, it is not necessary to use both maximum and minimum limit circuits. Only one of the limit circuits is required due to the symmetry of the sinewave that is centered around the virtual ground voltage at the non-inverting terminal of the amplifiers ($V_{DD}/2$). In the reference design of Figure 6, V_{DD} is equal to 5V and $V_{Q1\text{-}base}$ is equal to 2.5V. Thus, the oscillation waveform at V_2 will swing from 1.8V to 3.2V or 2.5V ±0.7V.

Note that the transistor adds a small capacitance (C_{Q1}) to the integrator capacitor of the circuit (C_2). If C_2 is relatively small, the effective capacitance of the limit circuit (C_{Limit}) can be reduced by connecting a diode in series between the emitter junction and the output of the amplifier (i.e. $1/C_{Limit} = 1/C_{Q1} + 1/C_{Diode}$).

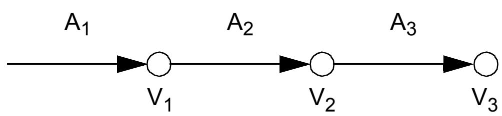
RATIO STATE-VARIABLE

Design Equations

STEP 1: FIND LG AND ΔS

$$T(s) = \frac{A}{1 - LG} = \frac{A}{\Delta s} = \frac{A}{\frac{N(s)}{D(s)}}$$

The loop-gain is found by breaking the loop in the signal flow diagram of Figure 7, as shown below.



$$\begin{aligned} A_1 &= -1/(sR_1C_1) \\ A_2 &= -1/(sR_2C_2) \\ A_3 &= Z_4/Z_3 \\ &= -[(R_4 \parallel C_4)/(R_3 + (R_5 \parallel C_3))] \\ &= -[R_4(sR_5C_3 + 1)/(sR_3R_5C_3 + R_3 + R_5)(sR_4C_4 + 1)] \end{aligned}$$

LG = A_1 x A_2 x A_3:

$$LG = [-1/(sR_1C_1)][-1/(sR_2C_2)]\left[\frac{[-R_4(sR_5C_3 + 1)]}{(sR_3R_5C_3 + R_3 + R_5)(sR_4C_4 + 1)}\right]$$

$$= \left[-(sR_4R_5C_3 + R_4)/s^4R_1R_2R_3R_4C_1C_2C_3C_4 + s^3((R_1R_2C_1C_2)(R_3R_5C_3 + R_3R_4C_4 + R_4R_5C_4)) + s^2(R_1R_2C_1C_2)(R_3 + R_5)\right]$$

Δs = N(s) / D(s) = 1 - LG:

$$\Delta s = \frac{\left[\left(s^4R_1R_2R_3R_4R_5C_1C_2C_3C_4 + s^3(R_1R_2C_1C_2)(R_3R_5C_3 + R_3R_4C_4 + R_4R_5C_4)\right) + s^2(R_1R_2C_1C_2)(R_3 + R_5) + sR_4R_5C_3 + R_4\right]}{\left[\left(s^4R_1R_2R_3R_4R_5C_1C_2C_3C_4\right) + s^3((R_1R_2C_1C_2)(R_3R_5C_3 + R_3R_4C_4 + R_4R_5C_4)) + s^2(R_1R_2C_1C_2)(R_3 + R_5)\right]}$$

STEP 2: SOLVE N(s) = 0

The zeroes of the characteristic equation are determined by using the Routh stability test:

$$\begin{aligned} Ns &= a_0s^4 + a_1s^3 + a_2s^2 + a_3s + a_4 \\ a_0 &= R_1R_2R_3R_4R_5C_1C_2C_4 \\ a_1 &= (R_1R_2C_1C_2)(R_3R_5C_3 + R_3R_4C_4 + R_4R_5C_4) \\ a_2 &= (R_1R_2C_1C_2)(R_3 + R_5) \\ a_3 &= R_4R_5C_3 \\ a_4 &= R_4 \end{aligned}$$

Routh Stability Test Coefficient Array

row s^4 $a_0 + a_2 + a_4 = 0$

row s^3 $a_1 + a_3 = 0$

Row s^4 produces an equation that cannot be solved with simple algebra. Therefore, the next row is analyzed:

$$a_0(j\omega_o)^4 + a_2(j\omega_o)^2 + a_4 = 0$$

The procedure continues by analyzing row s^3 to determine when the row equation is equal to zero.

$$a_1 s^3 + a_3 s = s\left(a_1 s^2 + a_3\right) = 0$$

Let $s = j\omega_o$

$$j\omega_o\left(-a_1\omega_o^2 + a_3\right) = 0$$

$$\omega_o^2 = \frac{a_3}{a_1}$$

$$= \frac{R_4R_5C_3}{(R_1R_2C_1C_2)(R_3R_5C_3 + R_3R_4C_4 + R_4R_5C_4)}$$

$$\omega_o = \left[\frac{(R_4R_5C_3)}{(R_1R_2C_1C_2)(R_3R_5C_3 + R_3R_4C_4 + R_4R_5C_4)}\right]^{1/2}$$

If:

1. $R_1 = R_2 = R$
2. $C_1 = C_2 = C$

Then:

$$P = \frac{2\pi}{\omega_o}$$

$$P = 2\pi RC\left[\frac{R_3C_4}{R_5C_3} + \frac{C_4}{C_3} + \frac{R_3}{R_4}\right]^{1/2}$$

If:

1. $R_5 >> R_3$
2. $R_4 >> R_3$

Then:

$$P \cong 2\pi RC\left[\frac{C_4}{C_3}\right]^{1/2}$$

STEP 3: SUB-CIRCUIT DESIGN EQUATIONS

Integrator A_1

$$Gain\ A_1 = -1/(2\pi f R_1 C_1)$$
$$Pole\ f = 1/(2\pi R_1 C_1)$$

Integrator A_2

$$Gain\ A_2 = -1/(2\pi f R_2 C_2)$$
$$Pole\ f = 1/(2\pi R_2 C_2)$$

Differentiator A_3

$$DC\ Gain = -R_4/(R_3 + R_5)$$
$$Gain\ at\ Oscillation = -\frac{C_3}{C_4}$$
$$Pole\ f_{p1} = 1/(2\pi R_4 C_4)$$
$$Pole\ f_{p2} = 1/(2\pi R_3 C_3)$$
$$Zero\ f_z = 1/(2\pi R_5 C_3)$$

STEP 4: VERIFY $|LG| \geq 1$

Assume:

1. $R_5 >> R_3$ and $R_4 >> R_3$, then $A_3 = -C_3 / C_4$
2. $V_2 = V_{Max\_Limit}$ (i.e. place limit circuit at A_2)

Next, calculate the voltages at the output of each amplifier starting at V_2.

$$V_2 = V_{Max\_Limit}$$
$$V_3 = |A_3 \times V_2| = \left|\frac{C_3}{C_4} \times V_{Max\_Limit}\right|$$
$$V_1 = |A_1 \times V_3| = \left|\frac{1}{2\pi f R_1 C_1} \times V_3\right|$$

Oscillation will be sustained if:

$$|A_2 \times V_1| \geq V_{Max\_Limit}$$
$$\left|\left[\left(\frac{1}{2\pi f R_2 C_2}\right) \times V_1\right]\right| \geq V_{Max\_Limit}$$

RATIO STATE-VARIABLE

Test Results

$R_1 = R_2 = R_6 = R_7 =$ 32.7 kΩ
$C_1 = C_2 =$ 220 pF
$C_3 = C_4 =$ see Table 2
$R_3 =$ 5 kΩ
$R_4 =$ 3.3 MΩ
$R_5 =$ 10 MΩ
$R_8 =$ 1 kΩ
$R_9 =$ 1 MΩ
$V_{DD} =$ 5.0V
$A_1, A_2, A_3 \equiv$ MCP6024 (quad RRIO, GBW = 10 MHZ)
$A_4 \equiv$ MCP6541 Push-Pull Output Comparator
$Q_1 \equiv$ 2N3906

The measured and calculated oscillation frequency is shown in Table 2. Figure 9 shows the oscillation waveform when C_3 = 100 pF and C_4 = 47 pF. Note that the voltage limit circuit adds distortion to the waveform of amplifier A_2. In most sensor applications, waveform distortion is inconsequential because the measurement is proportional to frequency and not the amplitude of the oscillation.

TABLE 2: RATIO OSCILLATOR TEST RESULTS

Capacitor Values	Calculated Oscillation Period (Frequency)	Measured Oscillation Period (Frequency)
C_3 = 47 pF C_4 = 47 pF	45.2 µs (22.1 kHz)	47.0 µs (21.3 kHz)
C_3 = 47 pF C_4 = 100 pF	65.9 µs (15.2 kHz)	68.0 µs (14.7 kHz)
C_3 = 47 pF C_4 = 220 pF	97.8 µs (10.2 kHz)	98.4 µs (10.2 kHz)
C_3 = 56 pF C_4 = 47 pF	41.4 µs (24.2 kHz)	43.0 µs (23.3 kHz)
C_3 = 56 pF C_4 = 220 pF	89.6 µs (11.2 kHz)	92.0 µs (10.9 kHz)
C_3 = 100 pF C_4 = 47 pF	31.0 µs (32.3 kHz)	33.0 µs (30.3 kHz)
C_3 = 100 pF C_4 = 220 pF	67.0 µs (14.9 kHz)	70.0 µs (14.3 kHz)

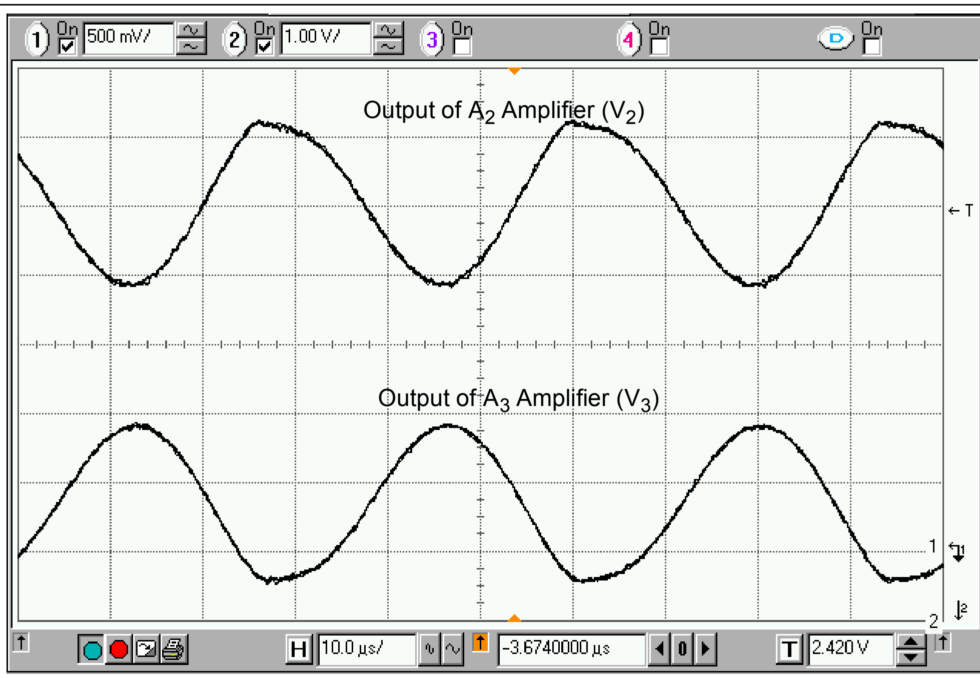


FIGURE 9: Ratio Oscillator Test Results (C_3 = 100 pF, C_4 = 47 pF).

APPENDIX A: MASON'S REDUCTION THEOREM

The oscillation frequency is determined by finding the poles of the denominator of the transfer equation T(s) or equivalently the zeroes of the numerator N(s) of the characteristic equation $\Delta(s)$. Mason's theory is especially useful for analyzing oscillators that have multiple feedback loops.

Mason's theorem [5] states that the transfer function from input X to output Y is:

EQUATION:

$$T(s) = \frac{Y}{X} = \frac{\Sigma_i P_i \Delta s_i}{\Delta s} = \frac{\Sigma_i P_i \Delta s_i}{\frac{N(s)}{D(s)}}$$

Where:

P_i = the direct transmittance or path form input X to output Y

Δs_i = the system determinant. (Δs_i = 1 if P_i touches all of the loops)

$\Delta s = 1 - \Sigma L_j + \Sigma L_k L_l - \Sigma L_m L_n L_o + \ldots$

ΣL_j = the sum of all loops (i.e. loop gains)

$\Sigma L_k L_l$ = the sum of products of pairs of non-touching loops

$\Sigma L_m L_n L_o$ = the sum of products of gains of non–touching loops taken three at a time.

APPENDIX B: ROUTH STABILITY TEST

The Routh Stability Test [5] can be used to test the characteristic equation to determine whether any of the roots lie on the imaginary axis. Routh's test consists of forming a coefficient array from N(s). Next, the procedure substitutes $s = j\omega_o$ for s, and the summation of the row is set to zero. If the row equation produces a non-trivial solution for ω_o, the procedure is complete and the frequency of oscillation is equal to ω_o. If the row equation does not yield an equation that can be solved for ω_o, the procedure continues with the next row in the Routh array. This technique arranges the numerator of the characteristic equation (i.e., denominator of the transfer equation) into the array listed below.

EQUATION:

$$N(s) = a_o s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \ldots + a_{n-1} s + a_n$$

Note for simplicity, only the first three rows of the Routh coefficient array are shown below.

$$\begin{array}{lllll} s^n & a_0 & a_2 & a_4 \ldots & a_n \\ s^{n-1} & a_1 & a_3 & a_5 \ldots & a_{n-1} \\ s^{n-2} & b_1 & b_2 & b_3 \ldots & b_{n-2} \end{array}$$

where the coefficients b_1, b_2, b_3, etc., are defined as:

$b_1 = (a_1 a_2 - a_0 a_3) / a_1$

$b_2 = (a_1 a_4 - a_0 a_5) / a_1$

$b_3 = (a_1 a_6 - a_0 a_7) / a_1$

The Routh stability criterion states:

1. A necessary and sufficient condition for stability is that the first column of the array does not contain sign changes.
2. The number of sign changes in the entries of the first column of the array is equal to the number of roots in the right half s–plane.
3. If the first element in a row is zero, it is replaced by ε, and the sign changes when $\varepsilon \to 0$ are counted after completing the array.
4. The poles are located in the right half plane or on the imaginary axis if all the elements in a row are zero.

CONCLUSION

Operational amplifier oscillators can be used to produce a frequency that is proportional to resistive and capacitive sensors. Design equations defining the oscillation frequency are readily available for several common oscillators, such as Wein bridge and phase shift oscillators. However, detailed design equations that show the relationship of the resistors and capacitors are generally not available. Thus, there is a need for a design procedure that derives the equations in order to select the resistor and capacitor components that maximize the accuracy of the oscillation frequency. The design procedure was demonstrated by analyzing two state-variable oscillators for capacitive sensing applications.

BIBLIOGRAPHY

1. Celma, C., Martinez, P., and Carlosens, A., “Approach to the Synthesis of Canonic RC–Active Oscillators Using CCII”, IEE Proc. Circuits, Devices and Systems, Vol. 141, No. 6, December 1994, pp. 493–497.
2. Lepkowski, J. and Young, C, “AND8054 - Designing RC Oscillator Circuits with Low Voltage Operational Amplifiers and Comparators for Precision Sensor Applications”, ON Semiconductor, Phoenix, Arizona, 2002.
3. Martinez, P., Aldea C. and Celma, S., “Approach to the Realization of State-Variable Based Oscillators”, IEEE International Conference on Electronics Circuits and Systems, Vol. 3, 1998, p.139–142.
4. Sidorowicz, R., “An Abundance of Sinusoidal RC–Oscillators”, Proc. IEE, Vol. 119, No. 3, March 1972, pp. 283–293.
5. Truxal, J., “Introductory System Engineering”, McGraw–Hill, N.Y., 1972.
6. Van Valkenburg, M., “Analog Filter Design”, Saunders College Publishing, Fort Worth, 1992.

NOTES:

WORLDWIDE SALES AND SERVICE

AMERICAS

Corporate Office
2355 West Chandler Blvd.
Chandler, AZ 85224-6199
Tel: 480-792-7200 Fax: 480-792-7277
Technical Support: 480-792-7627
Web Address: http://www.microchip.com

Atlanta
3780 Mansell Road, Suite 130
Alpharetta, GA 30022
Tel: 770-640-0034 Fax: 770-640-0307

Boston
2 Lan Drive, Suite 120
Westford, MA 01886
Tel: 978-692-3848 Fax: 978-692-3821

Chicago
333 Pierce Road, Suite 180
Itasca, IL 60143
Tel: 630-285-0071 Fax: 630-285-0075

Dallas
4570 Westgrove Drive, Suite 160
Addison, TX 75001
Tel: 972-818-7423 Fax: 972-818-2924

Detroit
Tri-Atria Office Building
32255 Northwestern Highway, Suite 190
Farmington Hills, MI 48334
Tel: 248-538-2250 Fax: 248-538-2260

Kokomo
2767 S. Albright Road
Kokomo, IN 46902
Tel: 765-864-8360 Fax: 765-864-8387

Los Angeles
18201 Von Karman, Suite 1090
Irvine, CA 92612
Tel: 949-263-1888 Fax: 949-263-1338

Phoenix
2355 West Chandler Blvd.
Chandler, AZ 85224-6199
Tel: 480-792-7966 Fax: 480-792-4338

San Jose
Microchip Technology Inc.
2107 North First Street, Suite 590
San Jose, CA 95131
Tel: 408-436-7950 Fax: 408-436-7955

Toronto
6285 Northam Drive, Suite 108
Mississauga, Ontario L4V 1X5, Canada
Tel: 905-673-0699 Fax: 905-673-6509

ASIA/PACIFIC

Australia
Microchip Technology Australia Pty Ltd
Marketing Support Division
Suite 22, 41 Rawson Street
Epping 2121, NSW
Australia
Tel: 61-2-9868-6733 Fax: 61-2-9868-6755

China - Beijing
Microchip Technology Consulting (Shanghai) Co., Ltd., Beijing Liaison Office
Unit 915
Bei Hai Wan Tai Bldg.
No. 6 Chaoyangmen Beidajie
Beijing, 100027, No. China
Tel: 86-10-85282100 Fax: 86-10-85282104

China - Chengdu
Microchip Technology Consulting (Shanghai) Co., Ltd., Chengdu Liaison Office
Rm. 2401-2402, 24th Floor,
Ming Xing Financial Tower
No. 88 TIDU Street
Chengdu 610016, China
Tel: 86-28-86766200 Fax: 86-28-86766599

China - Fuzhou
Microchip Technology Consulting (Shanghai) Co., Ltd., Fuzhou Liaison Office
Unit 28F, World Trade Plaza
No. 71 Wusi Road
Fuzhou 350001, China
Tel: 86-591-7503506 Fax: 86-591-7503521

China - Hong Kong SAR
Microchip Technology Hongkong Ltd.
Unit 901-6, Tower 2, Metroplaza
223 Hing Fong Road
Kwai Fong, N.T., Hong Kong
Tel: 852-2401-1200 Fax: 852-2401-3431

China - Shanghai
Microchip Technology Consulting (Shanghai) Co., Ltd.
Room 701, Bldg. B
Far East International Plaza
No. 317 Xian Xia Road
Shanghai, 200051
Tel: 86-21-6275-5700 Fax: 86-21-6275-5060

China - Shenzhen
Microchip Technology Consulting (Shanghai) Co., Ltd., Shenzhen Liaison Office
Rm. 1812, 18/F, Building A, United Plaza
No. 5022 Binhe Road, Futian District
Shenzhen 518033, China
Tel: 86-755-82901380 Fax: 86-755-8295-1393

China - Qingdao
Rm. B505A, Fullhope Plaza,
No. 12 Hong Kong Central Rd.
Qingdao 266071, China
Tel: 86-532-5027355 Fax: 86-532-5027205

India
Microchip Technology Inc.
India Liaison Office
Marketing Support Division
Divyasree Chambers
1 Floor, Wing A (A3/A4)
No. 11, O'Shaugnessey Road
Bangalore, 560 025, India
Tel: 91-80-2290061 Fax: 91-80-2290062

Japan
Microchip Technology Japan K.K.
Benex S-1 6F
3-18-20, Shinyokohama
Kohoku-Ku, Yokohama-shi
Kanagawa, 222-0033, Japan
Tel: 81-45-471- 6166 Fax: 81-45-471-6122

Korea
Microchip Technology Korea
168-1, Youngbo Bldg. 3 Floor
Samsung-Dong, Kangnam-Ku
Seoul, Korea 135-882
Tel: 82-2-554-7200 Fax: 82-2-558-5934

Singapore
Microchip Technology Singapore Pte Ltd.
200 Middle Road
#07-02 Prime Centre
Singapore, 188980
Tel: 65-6334-8870 Fax: 65-6334-8850

Taiwan
Microchip Technology (Barbados) Inc.,
Taiwan Branch
11F-3, No. 207
Tung Hua North Road
Taipei, 105, Taiwan
Tel: 886-2-2717-7175 Fax: 886-2-2545-0139

EUROPE

Austria
Microchip Technology Austria GmbH
Durisolstrasse 2
A-4600 Wels
Austria
Tel: 43-7242-2244-399
Fax: 43-7242-2244-393

Denmark
Microchip Technology Nordic ApS
Regus Business Centre
Lautrup hoj 1-3
Ballerup DK-2750 Denmark
Tel: 45-4420-9895 Fax: 45-4420-9910

France
Microchip Technology SARL
Parc d'Activite du Moulin de Massy
43 Rue du Saule Trapu
Batiment A - ler Etage
91300 Massy, France
Tel: 33-1-69-53-63-20 Fax: 33-1-69-30-90-79

Germany
Microchip Technology GmbH
Steinheilstrasse 10
D-85737 Ismaning, Germany
Tel: 49-89-627-144-0
Fax: 49-89-627-144-44

Italy
Microchip Technology SRL
Via Quasimodo, 12
20025 Legnano (MI)
Milan, Italy
Tel: 39-0331-742611 Fax: 39-0331-466781

United Kingdom
Microchip Ltd.
505 Eskdale Road
Winnersh Triangle
Wokingham
Berkshire, England RG41 5TU
Tel: 44-118-921-5869 Fax: 44-118-921-5820

05/30/03